## The Van Meegeren Art Forgeries

After the liberation of Belgium in World War II, the Dutch Field Security began its hunt for Nazi collaborators. They discovered, in the records of a firm which had sold numerous works of art to the Germans, the name of a banker who had acted as an intermediary in the sale to Goering of the painting Woman Taken in Adultery by the famed 17th century Dutch painter Jan Vermeer. The banker in turn revealed that he was acting on behalf of a third-rate Dutch painter H.A. Van Meegeren, and on May 29, 1945 Van Meegeren was arrested on the charge of collaborating with the enemy. On July 12, 1945 Van Meegeren startled the world by announcing from his prison cell that he had never sold Woman Taken in Adultery to Goering. Moreover, he stated that this painting and the very famous Disciples at Emmaus as well as four other presumed Vermeers and two de Hooghs were his own works. Many people thought that Van Meegeren was lying to save himself from the charge of treason. To prove his point, Van Meegern began, while in prison, to forge the Vermeer painting Jesus Amongst the Doctors to demonstrate to the skeptics just how good a forger of Vermeer he really was. The work was nearly completed when Van Meegeren learned that a charge of forgery had been substituted for that of collaboration. He therefore refused to finish and age the painting in the hope that investigators would not uncover his secret of aging his forgeries. To settle the question, an international panel of distinguished chemists, physicists and art historians was appointed to investigate the matter. The panel took x-rays of the paintings to determine whether other paintings were underneath them. In addition, they analyzed the pigments (coloring materials) used in the paint and examined the paintings for certain signs of age.

Van Meegeren was well aware of these methods. To avoid detection, he scraped the paint from old paintings that were not worth much just to get the canvas, and he tried to use pigments that Vermeer would have used. Van Meegeren also knew that old paint was extremely hard and impossible to dissolve. Therefore, he cleverly mixed a chemical (phenoformaldehyde) into his paint, and this hardened into Bakelite when the finished painting was heated in an oven.

However, Van Meegeren was careless with several of his forgeries, and the panel of experts found traces of the modern pigment cobalt blue. In addition, they also detected the phenoformaldehyde (which was first discovered at the close of the 19th century) in several of the paintings. On the basis of the evidence Van Meegeren was convicted on October 12, 1947 and sentenced to one year in prison. While in prison he suffered a heart attack and died on December 30, 1947.

Despite the evidence gathered by the panel of experts, many people still refused to believe that the famed *Disciples at Emmaus* was forged by Van Meegeren. Their contention was based on the fact that the other alleged forgeries and Van Meegeren's nearly completed *Jesus Amongst the Doctors* were of a very inferior quality. Surely, they said, the creator of the beautiful *Disciples at Emmaus* could not produce such inferior pictures. Indeed, the *Disciples at Emmaus* was certified as an authentic Vermeer by the noted art historian A. Bredius and was bought by the Rembrandt Society for \$170,000. The answer of the panel to these skeptics was that because Van Meegeren was keenly disappointed by his lack of status in the art world, he worked on the *Disciples at Emmaus* with the fierce determination of proving that he was better than a third-rate painter. After producing such a masterpiece his determination was gone. Moreover, after seeing how easy it was to dispose of the *Disciples at Emmaus* he devoted less effort to his subsequent forgeries. This explanation failed to satisfy the skeptics. They demanded a thoroughly scientific and conclusive proof that the *Disciples at Emmaus* was indeed a forgery. This was done in 1967 by scientists at Carnegie Mellon University and we will now describe their work.

The key to the dating of paintings and other material such as rocks and fossils lies in the phenomenon of radioactivity discovered at the turn of the century. The physicist Rutherford and his colleagues showed that the atoms of certain radioactive elements are unstable and that within a given time period a fixed proportion of the atoms spontaneously disintegrates to form atoms of a new element. Because radioactivity is a property of the atom, Rutherford showed that the radioactivity of a substance is directly proportional to the number of atoms of the substance present. Thus, if N(t) denotes the number of atoms present at time t, then dN/dt denotes the number of atoms that disintegrate per unit time and is proportional to N, that is

$$\frac{dN}{dt} = -\lambda N \,.$$

Here  $\lambda > 0$  is known as the decay constant of the substance. The larger  $\lambda$  is, of course, the faster the substance decays. One measure of the rate of disintegration of a substance is its half-life which is defined as the time required for half of a given quantity of radioactive atoms to decay. To compute the half-life of a substance in terms of  $\lambda$ , assume that at time  $t_0$ ,  $N(t_0) = N_0$ .

1. (a.) Analytically find the solution to the initial value problem

$$\frac{dN}{dt} = -\lambda N$$
$$N(t_0) = N_0$$

Note that the equation is separable.

(b) Use the command **DSolve** in *Mathematica* to find the general solution of this differential equation. After you have found the general solution, then use **DSolve** again to find the particular solution which satisfies the given initial condition. Make sure that this agrees with your answer in (a).

2. Show analytically that the half-life (in years) of a substance is  $\ln 2$  divided by the decay constant  $\lambda$ . Then use the **Solve** command to verify.

Now the basis of radioactive dating is essentially the following. From above, we can solve for  $t - t_0$ and if  $t_0$  is the time the substance was initially formed or manufactured, then the age of the substance is given by  $t - t_0$ . The decay constant  $\lambda$  is known or can be computed in most instances. Moreover we can usually evaluate N quite easily. Thus if we knew  $N_0$ , we could determine the age of the substance, but this is usually not possible. In some instances though, we can either determine  $N_0$  indirectly, or else determine certain suitable ranges for  $N_0$  and such is the case for the forgeries of Van Meegeren.

We begin with the following well-known facts of elementary chemistry. Almost all rocks in the earth's crust contain a small quantity of uranium. The uranium in the rock decays to another radioactive element, and that one decays to another, and so forth, in a series of elements that results in lead which is not radioactive. The uranium (whose half-life is over 4 billion years) keeps feeding the elements following it in the series, so that as fast as they decay, they are replaced by the elements before them.

Now, all paintings contain a small amount of the radioactive element lead-210 and an even smaller amount of radium-226. These elements both occur in white lead which is a pigment that artists have used for over 2000 years. For the analysis which follows, it is important to note that white lead is made from lead metal which, in turn, is extracted from a rock called lead ore in a process call smelting. In this process, the lead-210 in the ore goes along with the lead metal. However, 90-95% of the radium and its descendants are removed with other waste products in a material called slag. Thus most of the supply of lead 210 is cut off and it begins to decay very rapidly, with a half-life of 22 yr. This process continues until the lead-210 in the white lead is once more in radioactive equilibrium with the small amount of radium present, i.e., the disintegration of lead-210 is exactly balanced by the disintegration of radium.

Now we want to use this information to compute the amount of lead-210 present in a sample in terms of the amount originally present at the time of manufacture. Let y(t) be the amount of lead-210 per gram of white lead at time t,  $y_0$  the amount of lead-210 per gram of white lead present at the time of manufacture  $t_0$  and r(t) the number of disintegrations of radium-226 per minute per gram of white lead at time t. Let  $\lambda$  denote the decay constant for lead-210. Then we have

$$\frac{dy}{dt} = -\lambda y + r(t)$$

Since we are only interested in a time period of at most 300 years we assume that the radium-226, whose half-life is 1600 years, remains constant so that we replace r(t) by a constant r to obtain

$$\frac{dy}{dt} = -\lambda y + r$$
$$y(t_0) = y_0$$

- 3. (a) Analytically solve this differential equation. Hint: use an integrating factor.
  - (b) Use *Mathematica* to solve this initial value problem. Make sure your answer agrees with (a).

Now y(t) and r can be easily measured. Thus, if we knew  $y_0$  we could use your solution to compute  $(t-t_0)$  and consequently, we could determine the age of the painting. As we pointed out, though, we cannot measure  $y_0$  directly. One possible way out of this difficulty is to use the fact that the original quantity of lead-210 was in radioactive equilibrium with the larger amount of radium-226 in the ore from which the metal was extracted. Let us, therefore, take samples of different ores and compute the rate of disintegration of radium-226. This was done for a variety of ores and the results vary from 0.18 to 140. Consequently, the number of disintegrations of the lead-210 per minute per gram of white lead at the time of manufacture will vary from 0.18 to 140. This implies that  $y_0$  will also vary over a large interval since the number of disintegrations of lead-210 is proportional to the amount present. Thus we cannot use your result in (4) to obtain an accurate – or even a crude – estimate of the age of a painting. However, we can still use your result in (4) to distinguish between a 17th century painting and a modern forgery. The basis for this statement is the simple observation that if the painting is very old compared to the 22 year half-life of lead, then the amount of radioactivity from the lead-210 in a sample of paint will be nearly equal to the amount of radioactivity from the lead-210 will be much greater than the amount of radioactivity from the radium.

We can make this argument precise in the following manner. Let us assume that the painting in question is either very new or about 300 years old.

4. Set  $t - t_0 = 300$  in your solution in (3). Solve this expression for  $\lambda y_0$  to get

$$\lambda y_0 = \lambda y(t) e^{300\lambda} - r(e^{300\lambda} - 1)$$

Define a function in *Mathematica* for  $\lambda y_0$  (the number of disintegrations of the lead-210 per minute per gram of white lead at the time of manufacture) which has as its argument two parameters -  $\lambda y$  and r. Here  $\lambda y(t)$  represents the present disintegration rate of the lead-210 and r represents the disintegration rate of the radium-226.

If our painting is indeed a modern forgery, then  $\lambda y_0$  should be absurdly large. To determine what is an absurdly high disintegration rate we observe that if the lead-210 in a sample of white lead decays originally (at the time of manufacture) at the rate of 100 disintegrations then the ore from which it was extracted had a uranium content of 0.014. This is a fairly high concentration of uranium since the average amount of uranium in rocks of the earth's crust is about 2.7 parts per million. On the other hand, some very rare ores exist in the western hemisphere whose uranium content is 2-3%. To be on the safe side, we will say that a disintegration rate of lead-210 is certainly absurd if it exceeds 30,000 dis/min per gram of white lead.

To evaluate  $\lambda y_0$ , which is the number of disintegrations of the lead-210 per minute per gram of white lead at the time of manufacture, we must evaluate the present disintegration rate  $\lambda y(t)$  of the lead-210 and the disintegration rate r of the radium-226. Since the disintegration rate of polonium-210 equals that of lead-210 after several years and since it is easier to measure the disintegration rate of polonium-210, we substitute these values for those of lead-210. The disintegration rates of polonium-210 and radium-226 were measured for the *Disciplies at Emmaus* and various other alleged forgeries and are given in the table below.

Painting	PO-210 disintegration	Ra-226 disintegration
Disciplies at Emmaus	8.5	0.8
Washing of Feet	12.6	0.26
Lace Maker	1.5	1.4
Woman Reading Music	10.3	0.3
Woman Playing Mandolin	8.2	0.17
Laughing Girl	5.5	6.0

5. Determine which of the paintings are forgeries. To do this, use the function you defined in *Mathematica* and evaluate it using the values given for each of the paintings in the table. For  $\lambda$  use the half life of lead and (2).

On one graph, plot the value of the number of disintegrations of lead-210 per minute per gram of white lead at the time of manufacture for each painting. On the same plot, graph the threshold value which we are assuming. Also label each point with the name of the painting which it represents.

You may see if your answers agree with the experts by looking on the internet.