NUMERICAL METHODS FOR THE SOLUTION OF THE SHALLOW-WATER EQUATIONS IN METEOROLOGY

Abstract (summary)
This thesis deals with the system of partial differential equations representing motion in a homogeneous, incompressible and non-viscous fluid in hydrostatic equilibrium with a free surface. This system, referred to in meteorology as the shallow-water equations system or the primitive barotropic equations system, provides an approximation to large-scale weather prediction equations having the same numerical characteristics.

Our principal aim is to develop new methods for solving this system with improved accuracy or high computational efficiency. Several numerical aspects of this system of quasilinear hyperbolic partial differential equations, viewed as a mixed initial-boundary value problem, are highlighted. We show that the long-term behaviour of the shallow-water equations systems in the inviscid limit is related to the conservation of a number of integral invariants, in particular conservation of potential enstrophy.

We also show the paramount importance of well-posed numerical boundary conditions as a prerequisite for the success of limited-area integrations of the shallow-water equations systems.

The first chapter consists of a general introduction to the topic of numerical weather forecasting, in which are also introduced some of the basic mathematical tools and concepts which will be used at a later stage when the numerical methods are formulated.

Chapter 2 is devoted to a survey of the properties of the shallow-water equations system. Various difference approximations for the system are presented. Well-posedness of boundary conditions for the system on a limited-area domain is discussed for both the inviscid and the viscous cases. A proof is provided of uniqueness of solutions for well-posed boundary conditions.

A new time-differencing scheme which is particularly suitable for simulating the process of geostrophic adjustment is formulated and experimented with in Chapter 3. Numerical experiments with the shallow-water equations are performed both in a channel and on a nested fine-mesh grid, in combination with time-dependent boundary conditions derived from a coarse-mesh integration.

In Chapter 4 we propose a new linear ADI scheme with good stability properties by means of which high-accuracy solutions of the shallow-water equations can be achieved with high computational efficiency. Particular attention was devoted to the long-term behaviour of the shallow-water equations in the inviscid limit. The occurrence of catastrophic behaviour was found to be related to the theory of three-dimensional turbulence. The influence of critical dissipativity terms and the role of the conservation of potential enstrophy turned out to be crucial for obtaining numerical methods which are dynamically consistent with the original equations.

For the numerical solution of the shallow-water equations, a new finite-element scheme with a generalized mass-matrix formulation is proposed in Chapter 5. This method achieves high accuracy and a good conservation of integral invariants of the shallow-water equations for long-term integrations. The stability and accuracy of the method are compared with experimental results obtained by other investigators, and an explanation is offered for the high accuracy achieved by means of the generalized mixed mass finite-element scheme.

In Chapter 6 is derived a fourth-order compact implicit ADI method for solving the shallow-water equations system in conservation-law form. This method achieves fourth-order accuracy while preserving a compact discretization. Adequate numerical boundary conditions and dissipative terms for controlling aliasing are discussed. Numerical experiments confirm the fourth-order accuracy of this method. Finally, the connection is pointed out between the fourth-order compact method and the generalized mixed mass method for regular grids.
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