A Note on the Particle Filter with Posterior Gaussian Resampling

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ABSTRACT

Particle filter (PF) is a fully nonlinear filter with Bayesian conditional probability estimation, compared with the well known ensemble Kalman filter (EnKF). A Gaussian resampling (GR) method is proposed to generate the posterior analysis ensemble in an effective and efficient way. The Lorenz model is used to test the proposed method. The particle filter with Gaussian resampling (PFGR) can approximate more accurately the Bayesian analysis. The present work demonstrates that the proposed PFGR possesses good stability and accuracy and is potentially applicable to large-scale data assimilation problems.

1 INTRODUCTION

In recent years the ensemble filtering method has been the focus of increased interest in the meteorological community. The ensemble Kalman Filter (EnKF) (see review by Evensen, 2003) combines ensemble sampling and integration with Kalman filtering method, providing an approximated least square estimation of underlying physical states based on Monte Carlo sampling theory.

EnKF has been shown to be equivalent to the mean or maximal mode estimation of the posterior analysis under the assumption of linearized dynamics and observations based on Bayesian’s theory (see derivation by Cohn, 1997). It’s well known that, through direct evaluation of the Bayesian’s formula at each prior sample point, a particle filter (PF) generates a probability-weighted posterior sample. The evaluation does not restrict the probability distribution of the prior sample and the observation to be Gaussian. However, the probability weights are computed based on the observation which normally has no correlation with the dynamics. Therefore the resulting weighted sample is unlikely to provide an efficient sampling of a continuous probability distribution like a standard Monte Carlo sampling. In a sequential application the estimation error increases as the filter applies at every step. A large enough estimation error can induce a so-called filter divergence or degeneracy problem, which refers to the fact that the ensemble sample diverges gradually from the true state and no longer produces a meaningful forecast.

Although the PF showed varied degree of success, filter divergence remains a major concern in realistic application of the PF. Covariance inflation is the most common technique to stabilize the PF (Anderson and Anderson, 1999; Whitaker and Hamill, 2002). Inflation factors are introduced to offset a tendency of the ensemble forecast to become underdispersive. The cause of such underdispersion can be attributed to a failure to adequately represent model error, rank deficiency in the forecast error covariance model or misspecification of observation errors aspects of the algorithm. One of the primary concerns regarding inflation factors is that they do not address the root cause of ensemble underdispersion - sub-optimality in the filter. The inflation factor as a tuning parameter is also model and observation dependent, which can pose an extra layer of uncertainty in error sensitive filter applications, e.g. model error estimation. Other PF relies on the intrinsic smoothing capability of the model where the model noise and the nonlinear interactions among the growing modes may produce enough chaotic behavior to recover lost degrees of freedom in particle filtering (van Leeuwen, 2003).

This note proposes an “a posteriori” Gaussian resampling (GR) method that aims to increase the stability of the particle filter and maintain the ensemble spread, while allowing for a potential generalization to higher-dimensional models. The rest of the paper is organized as follow: Section 2 introduces a particle filter with the posterior Gaussian resampling (PFGR). Section 3 presents simulation results of an numerical test of the method using the Lorenz model comparing PFGR and EnKF. Section 4 concludes the work and discusses directions of future research effort.

2 Particle filter in Bayesian Framework

Dynamical evolution of discretized physical systems are described by

\[ x_k = \mathcal{M}(x_{k-1}) + g(x_{k-1})\epsilon_{k-1}, \]  

(1)