Homework 20 (Use of L-BFGS-B for constrained minimization)

LBFGS-B is an algorithm for solving large scale optimization problems subject to simple bounds.

It is based on the gradient projection method and uses a limited memory BFGS matrix to approximate the Hessian of the objective function.

The problem is

\[ \min f(x) \]
subject to \( l \leq x \leq u \)

where \( l \) and \( u \) are the lower and upper bounds on the variables.

1. Run the executable x1.lbfgs with the example provided by Ciyou Zhu and Jorge Nocedal.
2. Original code available at:


The tar.gz code is available on blackboard along with the papers

3. Describe in some detail the method based on which the code works and the constrained and unconstrained minimization methods used.

4. Describe the algorithmic procedure employed.

5. Compare results using driver1.f with those using driver2.f with its executable x.lbfgsb2

Typical output:

Solving sample problem.
\( (f = 0.0 \text{ at the optimal solution}) \)

RUNNING THE L-BFGS-B CODE

* * *
Machine precision = 1.084E-19
N = 25 M = 5

At X0 0 variables are exactly at the bounds

At iterate 0 f= 3.46000E+03  |proj g| = 1.03000E+02
At iterate 1 f= 2.39769E+03  |proj g| = 6.50700E+01
At iterate 2 f= 1.43806E+02  |proj g| = 3.64039E+01
At iterate 3 f= 7.28161E+01  |proj g| = 2.29042E+01
At iterate 4 f= 1.60308E+01  |proj g| = 6.95409E+00
At iterate 5 f= 5.18725E+00  |proj g| = 9.05481E+00

Bad direction in the line search;
refresh the lbfgs memory and restart the iteration.

At iterate 6 f= 2.87083E+00  |proj g| = 7.13636E+00
At iterate 7 f= 1.79199E+00  |proj g| = 2.20353E+00
At iterate 8 f= 9.66509E-02  |proj g| = 1.08444E+00
At iterate 9 f= 6.89435E-02  |proj g| = 1.86879E+00
At iterate 10 f= 1.03044E-02 |proj g| = 3.99596E-01

Bad direction in the line search;
refresh the lbfgs memory and restart the iteration.

At iterate 11 f= 6.72864E-03  |proj g| = 2.41447E-01
At iterate 12 f= 4.73618E-03  |proj g| = 1.10894E-01
At iterate 13 f= 2.99421E-03  |proj g| = 1.27485E-01
At iterate 14 f= 6.85522E-04  |proj g| = 2.00284E-01

Bad direction in the line search;
refresh the lbfgs memory and restart the iteration.
At iterate 15  \( f = 3.24501 \times 10^{-04} \)  \( |\text{proj g}| = 6.74779 \times 10^{-02} \)
At iterate 16  \( f = 2.35628 \times 10^{-04} \)  \( |\text{proj g}| = 3.87544 \times 10^{-02} \)
At iterate 17  \( f = 9.81723 \times 10^{-05} \)  \( |\text{proj g}| = 1.61094 \times 10^{-02} \)
At iterate 18  \( f = 3.82676 \times 10^{-05} \)  \( |\text{proj g}| = 1.75894 \times 10^{-02} \)
At iterate 19  \( f = 1.27763 \times 10^{-05} \)  \( |\text{proj g}| = 2.02902 \times 10^{-02} \)

Bad direction in the line search; refresh the lbfgs memory and restart the iteration.

At iterate 20  \( f = 3.04469 \times 10^{-06} \)  \( |\text{proj g}| = 1.04260 \times 10^{-02} \)
At iterate 21  \( f = 2.00330 \times 10^{-06} \)  \( |\text{proj g}| = 2.38175 \times 10^{-03} \)
At iterate 22  \( f = 1.60261 \times 10^{-06} \)  \( |\text{proj g}| = 1.86650 \times 10^{-03} \)

Bad direction in the line search; refresh the lbfgs memory and restart the iteration.

At iterate 23  \( f = 1.40305 \times 10^{-06} \)  \( |\text{proj g}| = 3.26593 \times 10^{-03} \)
At iterate 24  \( f = 1.05369 \times 10^{-06} \)  \( |\text{proj g}| = 2.08653 \times 10^{-03} \)
At iterate 25  \( f = 1.26016 \times 10^{-07} \)  \( |\text{proj g}| = 2.04081 \times 10^{-03} \)
At iterate 26  \( f = 4.72616 \times 10^{-08} \)  \( |\text{proj g}| = 7.48498 \times 10^{-04} \)
At iterate 27  \( f = 2.09876 \times 10^{-08} \)  \( |\text{proj g}| = 4.48829 \times 10^{-04} \)
At iterate 28  \( f = 3.77637 \times 10^{-09} \)  \( |\text{proj g}| = 1.63165 \times 10^{-04} \)
At iterate 29  \( f = 6.83331 \times 10^{-10} \)  \( |\text{proj g}| = 8.36442 \times 10^{-05} \)

Bad direction in the line search; refresh the lbfgs memory and restart the iteration.

At iterate 30  \( f = 3.51318 \times 10^{-10} \)  \( |\text{proj g}| = 8.18690 \times 10^{-05} \)
At iterate 31  \( f = 2.15502 \times 10^{-10} \)  \( |\text{proj g}| = 3.41499 \times 10^{-05} \)
At iterate 32  f=  1.18762E-10    |proj g|=  3.30345E-05
At iterate 33  f=  2.73518E-11    |proj g|=  3.26781E-05
At iterate 34  f=  1.39133E-11    |proj g|=  2.47513E-05
At iterate 35  f=  3.56995E-12    |proj g|=  6.10832E-06

***

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

***

N   Tit  Tnf  Tnint  Skip  Nact     Projg        F
25   35   47  105    0     0   6.108E-06   3.570E-12
F =  3.56995087E-12

CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL

Cauchy        time 0.000E+00 seconds.
Subspace minimization time 0.000E+00 seconds.
Line search    time 0.000E+00 seconds.

Total User time 0.000E+00 seconds.