Homework No 10 (BFGS Quasi-Newton Method)

Consider the BFGS quasi Newton method:

\[ H_{k+1} = (I - \rho_k s_k y_k^T) H_k ((I - \rho_k y_k s_k^T) + \rho_k s_k s_k^T) \]

\[ s_k = x_{k+1} - x_k \]

\[ y_k = \nabla f_{k+1} - \nabla f_k \]  \hspace{1cm} (BFGS)

\[ \rho_k = \frac{1}{y_k^T s_k} \]

for the inverse Hessian approximation \( H_k \).

For the direct Hessian approximation \( B_k \) we have via Sherman Morrison – Woodbury formula:

\[ B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k} \]  \hspace{1cm} (BFGS)

Use the attached code conminf.f or write equivalent code by yourself and test it for the difficult Woods function:

\[ f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1 - x_3)^2 \]
\[ + 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1) \]

a) Determine the Hessian of Woods function and obtain its eigenvalues and condition number for the point \( x = (-3, -1, -3, -1)^T \)

b) Use accuracy \( \varepsilon = 10^{-3} \) and \( \varepsilon = 10^{-2} \) and compare results for starting points:

c) \( x = (-3, -1, -3, -1)^T \) and \( x = (-1.2, 1, 1.2, 1)^T \)

d) Plot in semi log function and gradient norm vs. the number of iterations

e) Flowchart code of Conminf.f and explain its line-search method.