Final Projects
MAP 5395

Due: 5 p.m., 4/29

General Guidelines The project can be strictly computational, strictly theoretical or a combination of both. Projects which are a combination will consist of summarizing a paper(s) or chapter from a book and some computational results. A strictly theoretical project must discuss the method, provide and prove error estimates, etc. A strictly computational project must clearly state the problem solved giving the weak problem and the discrete weak problem and various choices you made such as quadrature rules, etc. In all computations convergence studies must be provided and a discussion of the changes that were required for your code. Your project write-up should be 10-15 typed pages including figures. You must provide a list of references.

Some Sample Projects

1. *The h^p Finite Element Method* In the standard FEM one chooses a particular degree of polynomial to use and refines the mesh to obtain a better approximation. In the h^p method you fix the mesh size h and increase the degree of the polynomial. Ivo Babuska has several seminal papers in this area. This project can be totally theoretical (i.e., discussing method, error estimates, etc.) or a combination.

2. *Finite Element Method with Penalty Method* We have seen how to satisfy different boundary conditions whether they are essential or natural. In the penalty method a term is added to the weak problem to satisfy inhomogeneous boundary conditions. The method should be discussed and an error estimate proved. Then numerical computations should be done to illustrate that the theoretical results are obtained. See Ciarlet (The FEM for Elliptic Problems), page 143. Babuska also has a paper in *Mathematics of Computation*, Vol 73.

3. *Mixed Finite Element Methods* This would be a strictly theoretical project which discusses this type of method, gives error estimates, etc. A starting point is Douglas Arnold's paper "Mixed FEMs for Elliptic Problems" which is available on the web. Other sources should be used to provide error estimates, etc.

4. *Finite Element Method on non-rectangular domains* This would be an entirely computational project. Choose domains in two dimensions such as a polygonal region and use a software package such as Triangle (Jonathon Shewchuk) to generate the grid. At least two different domains should be chosen. You should discuss the software package you are using and how you interfaced the geometry data with your code. Convergence studies should be completed to determine rates of convergence.

5. *Finite Element Method on 3-rectangles* We have not done any computations in three dimensions. You can set up a time dependent problem on a rectangular grid and solve

it using trilinear and triquadratic elements. Convergence studies should be completed to determine rates of convergence. Discuss the modifications to your code.

6. *Solving a fourth order problem* For one computational project you can take a fourth order problem in one dimension and implement cubic splines to solve it. Discuss modifications to the code and provide convergence studies to determine rates of convergence. The weak and discrete weak forms of the equation should be derived. For another project you could discuss in detail and compare different C^1 elements in two and three dimensions on n-simplices and n-rectangles.

7. *Adaptive Finite Element Methods* Typically we choose a mesh and a set of basis functions to solve our problems. Adaptive procedures try to automatically refine, coarsen or relocate a mesh and/or adjust the basis to achieve a solution with a specified accuracy in an optimal fashion. This would be a theoretical project which discusses some approaches to adaptive FEMs.

8. *Your choice* You may choose a project that is of particular interest to you or related to your research. However you must submit a short description of the project to me for approval by 4/4.