Natural Inclusion of Boundary Points in a Mesh

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https://people.sc.fsu.edu/∼jburkardt/presentations/...
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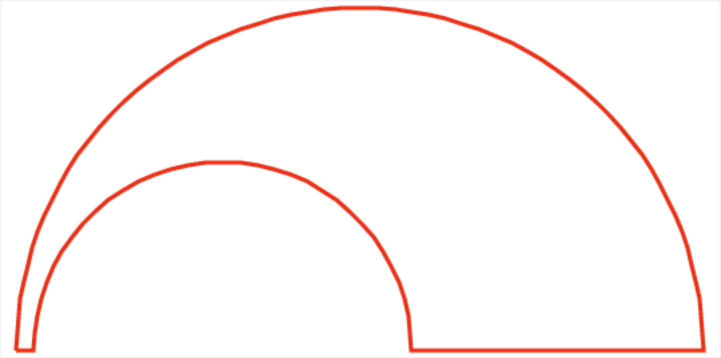
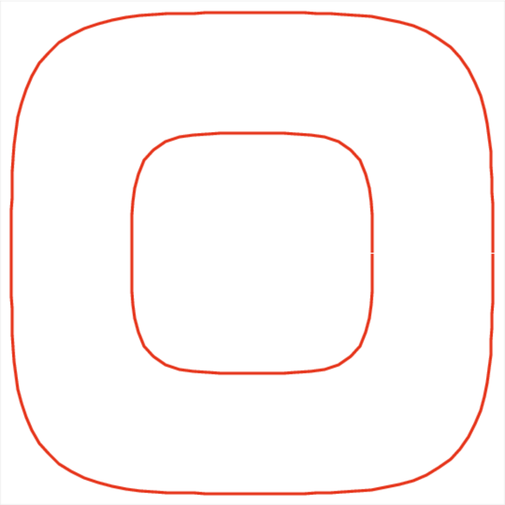
Meshes of Boundary and Interior

Given a “reasonable” region, compute a mesh as *automatically as possible, with good distribution in the interior and on the boundary.*

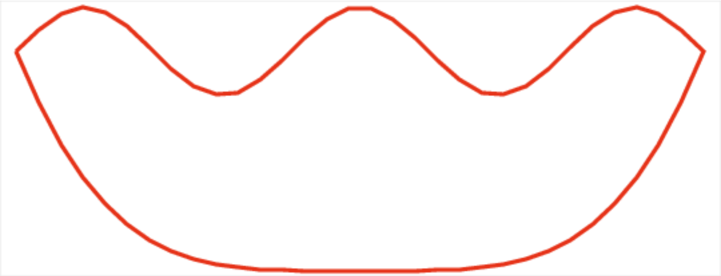
The Horn



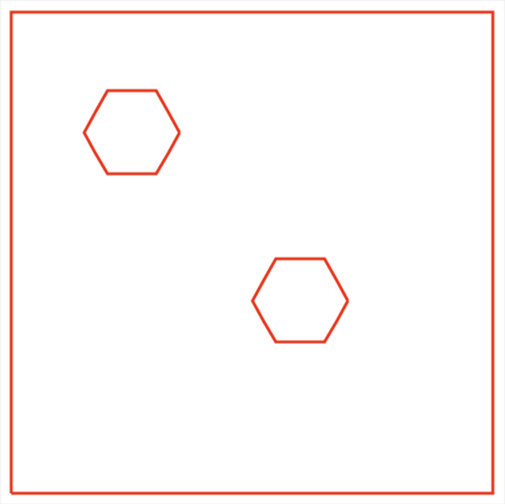
The Superellipse



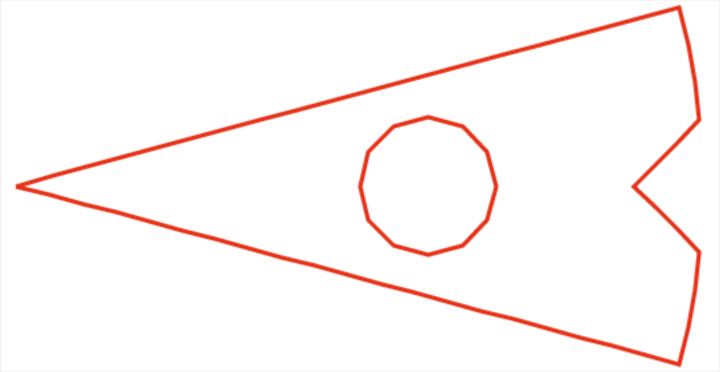
The Bicycle Seat



Two Hexagonal Holes



The "Holey" Pie



CVT

We are studying Centroidal Voronoi Tessellations. An iterative technique is available to approximate them.

- Initial set of points may be random;
- Compute Voronoi subregions;
- Replace each point by the centroid of its region;
- Iterate.

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Initialization

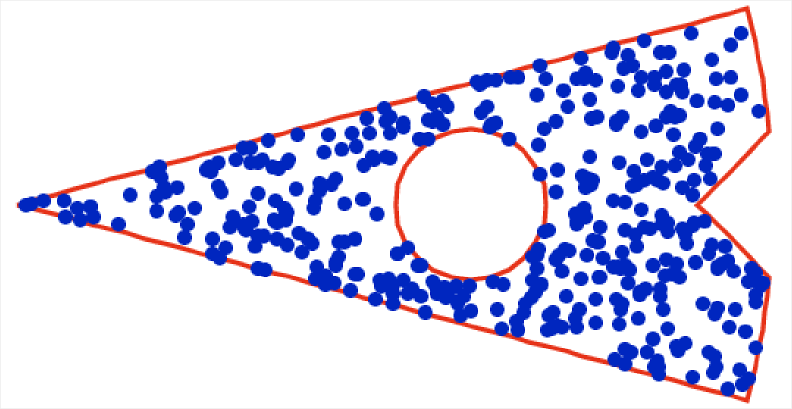


Figure: Initial points are random

CVT generators

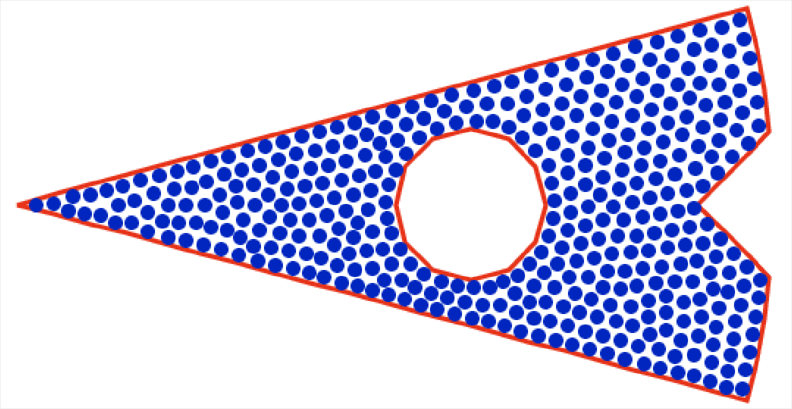


Figure: The converged generators

Can the CVT Algorithm be Adjusted?

We could simply move nearby points onto the boundary. But that would "tear" the mesh, disturbing the uniform distribution of points, the roughly equal size and equiangular shape of the triangles.

We could fix some points on the boundary, allowing them to slide around a bit. But choosing too many or too few will again disturb the mesh.

We need to "persuade" some points to approach the boundary, and give them a reason to stay there.

How Shewchuk's TRIANGLE Gets the Boundary

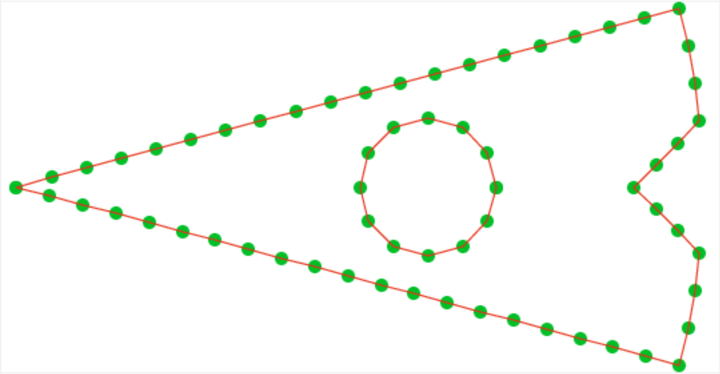


Figure: TRIANGLE requires "guide" nodes on the boundary

TRIANGLE works inward from Boundary Nodes

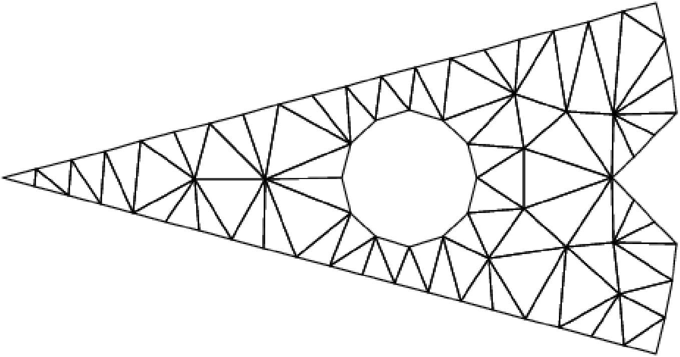


Figure: Mesh on boundary may be refined at points

A Finer TRIANGLE Mesh

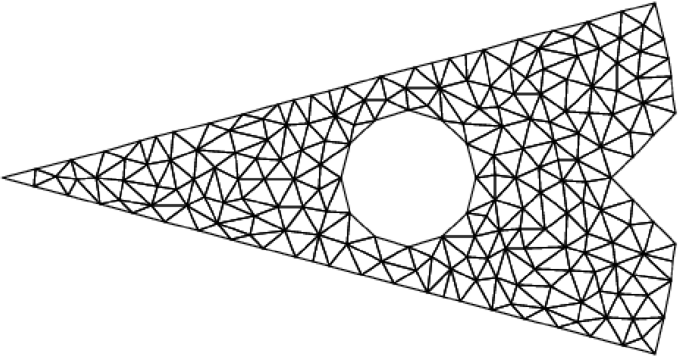


Figure: Triangles vary in shape and size

TRIANGLE's Mesh Generators May be Uneven

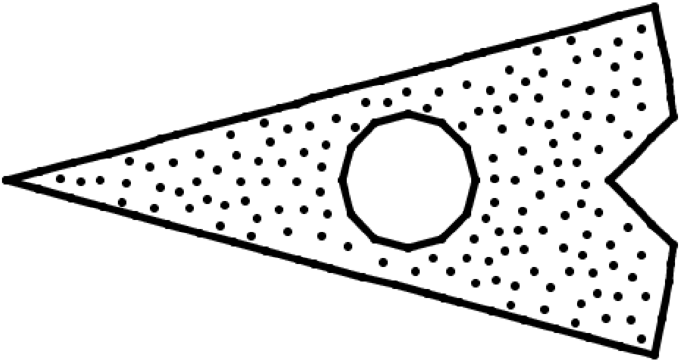


Figure: The points do not seem uniformly distributed

Persson and Strang's DISTMESH

- Compute a good mesh in the interior.
- Let it expand "a little" beyond the boundary.
- Push the exterior points back onto the boundary.
- Iterate this process so that it smooths out.

Initial DISTMESH Nodes

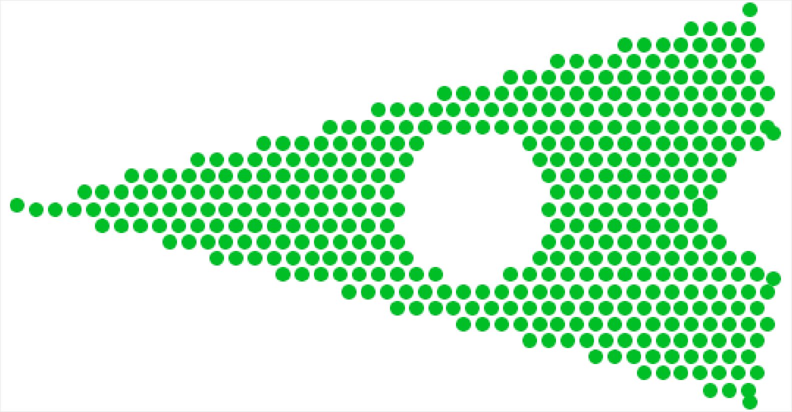


Figure: Hexagonal Lattice Data

Final DISTMESH Nodes and Mesh

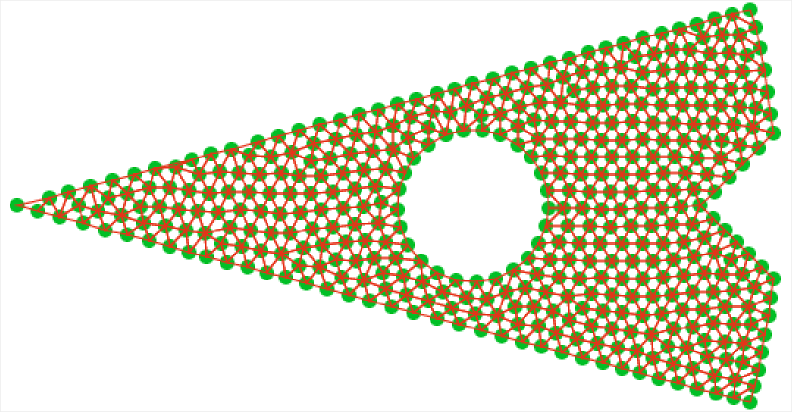


Figure: The derived mesh looks good

Final DISTMESH Nodes

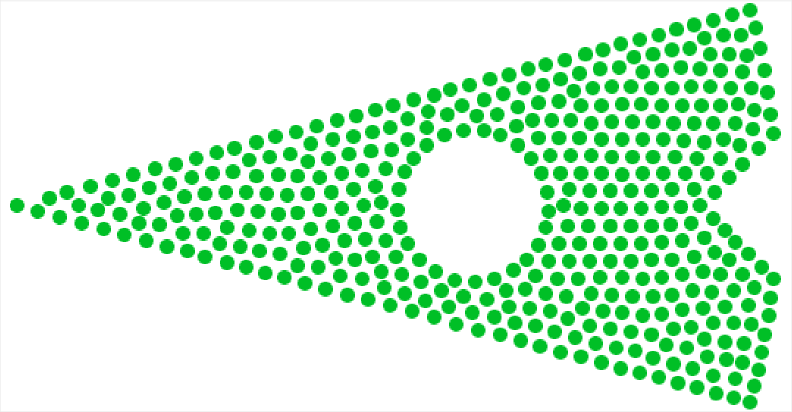


Figure: The nodes are uniformly distributed

Adapt DISTMESH Ideas to CVT

- Generate sample points in an expanded region.
- Assign the sample points to nearest generators, recompute centroids.
- Centroids exterior to the region get pushed back to the boundary.
- Iterate this process so that it smooths out.

CVT samples a thickened region

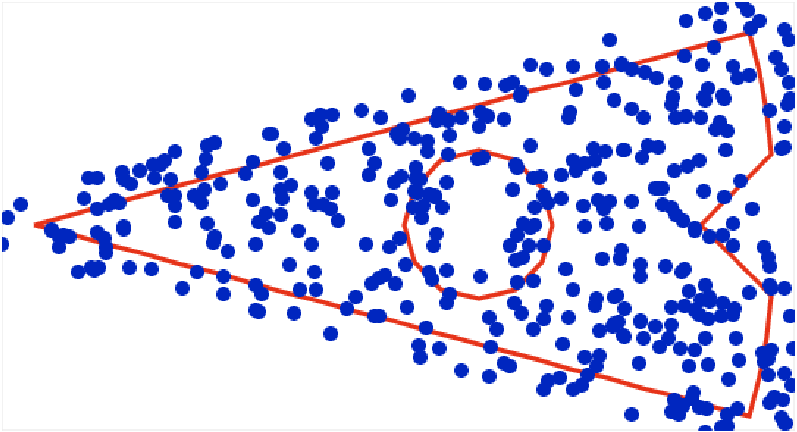


Figure: 400 samples

CVT generators

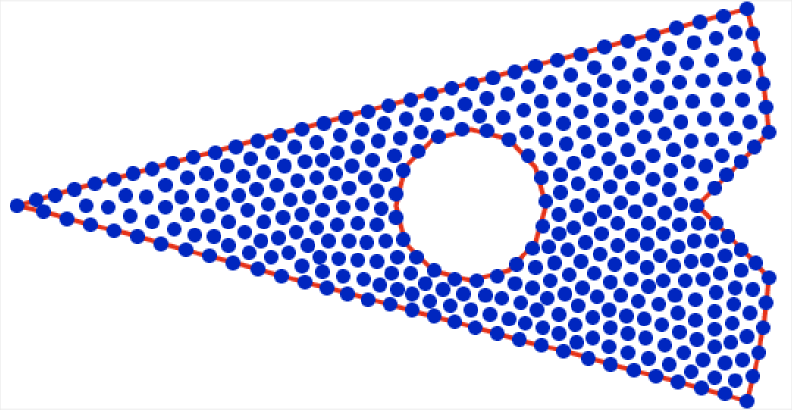
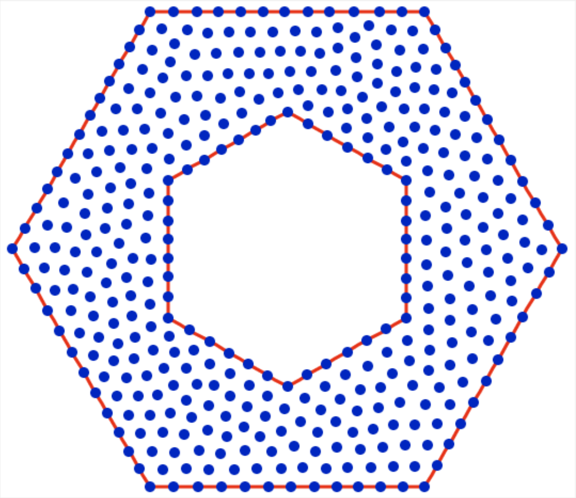


Figure: The converged generators

The "Hexnut"



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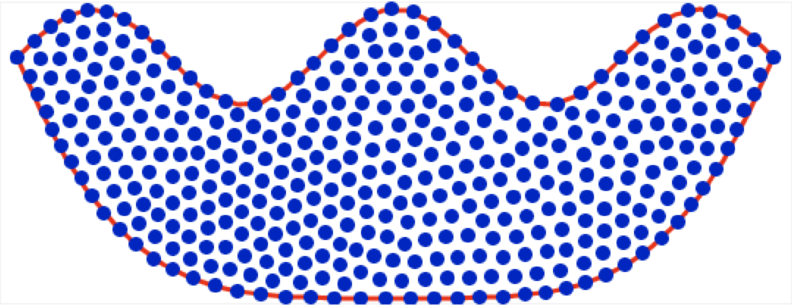
The "Bicycle Seat"



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Future Issues

- Problems in 3D can be handled in the same way;
- Nonuniform mesh density functions can be used;
- Voronoi regions could be computed exactly;
- Investigate relationship with Persson/Strang algorithm?
- Parameter-free methods would be preferred.

More Information

- Qiang Du, Vance Faber, Max Gunzburger, Centroidal Voronoi Tessellations, SIAM Review, December 1999.
- Per-Olof Persson and Gilbert Strang, A Simple Mesh Generator in MATLAB, SIAM Review, June 2004.
- Jonathan Shewchuk, Delaunay Refinement Algorithms for Triangular Mesh Generation, Computational Geometry, May 2002.