Computational Geometry Lab:
MISCELLANEOUS

John Burkardt
Information Technology Department
Virginia Tech
http://people.sc.fsu.edu/~jburkardt/presentations/cg_lab_miscellaneous.pdf

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1 Introduction

2 Some Definitions

By a **triangulation** we mean a collection of triangles.

Why this definition may seem sensible, we almost always implicitly assume more conditions on a triangulation.

Here, we will explicitly define a **regular triangulation** $\mathcal{T}$ to be a collection of triangles with the property that, when $i \neq j$, the intersection $T_i \cap T_j$ is either empty, or a single point which is a vertex of both triangles, or is a single line segment which is an entire edge of both triangles.

From now on, we will *always* assume that triangulations that we encounter or create are regular!

A **triangulation of a set of nodes** is a collection of triangles, each of which is defined by vertices taken only from the given set of nodes. Usually, it is required that every node in the set be used as a vertex at least once.

A **maximal triangulation of a set of nodes** is a regular triangulation of a set of nodes to which no further triangles can be added while preserving regularity. Essentially, this means that any newly created triangle will overlap the existing ones.

In some cases, our triangulation problem does not begin with a set of nodes, but with a region in the plane.

A **triangulation of a region** is a collection of triangles which exactly covers the region. A triangulation exactly covers a region if the union of the triangles is equal to the region. If a region is not polygonal, it cannot be covered by triangles. This would occur, for instance, if the region was a circle. In such cases, we may need to distinguish between the original region we are trying to approximate, and the actual (polygonal) region that is the union of the triangles.

If the region is polygonal, we might define it by listing the vertices of the polygonal line that bounds it. We might then add points along the boundary, and inside the region, and then triangulate these points. We must be careful, however, that the triangles we construct stay within the original region. Particularly if the region was not convex, there are guaranteed to be nodes that can be connected to form triangles that lie outside the region, which we do not want. Look at the last triangulation in Figure 3 for an example.
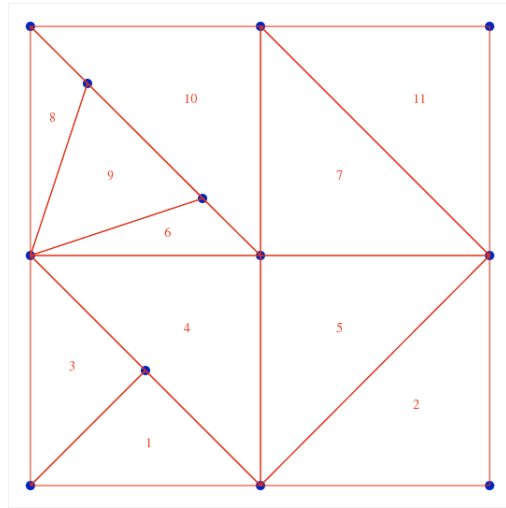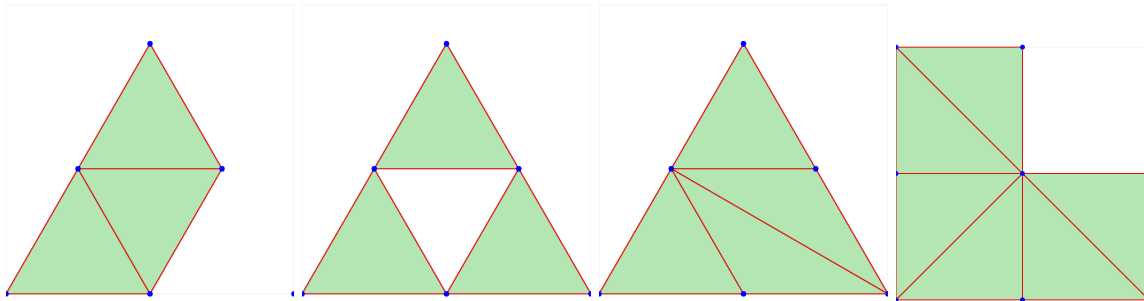
Figure 1: A triangulation that is not regular.

Figure 2: Four triangulations; only one is maximal. The green shading is only meant to suggest which triangles are included.

3 Program #6: Node Neighbors

4 The Quality of a Triangulation

Sometimes a triangulation needs to have certain properties because of the way it is going to be used. Depending on the situation, there are a number of *quality measures* that can report on how well a triangulation achieves a certain desired behavior. These quality measures are computed by a formula, and can be used to guide a program that is designing or modifying a triangulation.

It is a common convention that a perfect quality rating is 1, while the worst is 0.

Three simple quality measures are

- the α quality, the minimum angle over all triangles, divided by 60 degrees;

- the area quality, the ratio of the smallest and largest triangle areas;

- the Q quality, the minimum over all triangles of 2 * inradius / outradius.

Here, the "inradius" and "outradius" are the radiuses of the inscribed and circumscribed circles; for an equilateral triangle, these are in the ratio of $\frac{1}{2}$. Values of Q that are less than 1 indicate that some triangles are "deformed".

Note that the α and Q measures don't directly concern themselves with the issue of triangle size, and the area measure doesn't care about shape. If all three quality measures are good for a given triangulation, say at least 0.5, that can be a simple check that nothing is seriously wrong.

5 Program #7: The Quality of a Triangulation

Write a program which reads in a triangulation. evaluates the three quality measures and prints them.

What are the quality ratings for "house3"? For "hex_holes3". For "shrink3"?