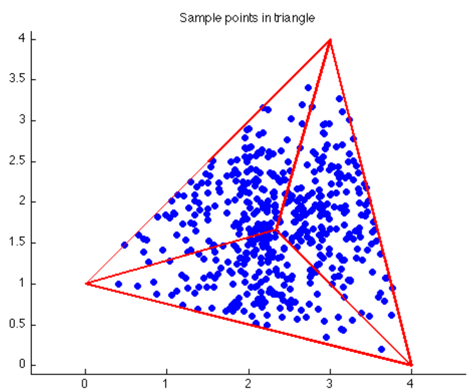
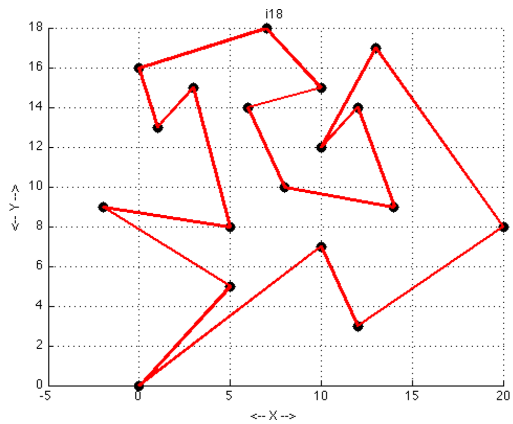
https://people.sc.fsu.edu/~jburkardt/presentations/asa_geometry_lab_2011_fsu.pdf

Algorithms II - Lab 8:
ALGORITHMS FOR GEOMETRY

Since we only have one lab session for computational geometry, please only do problems 1 and 2!





Problem 1: *Random Samples of a Triangle*

If α, β, γ are nonnegative and sum to 1, and A, B, C are the vertices of a triangle, then the point $p = \alpha A + \beta B + \gamma C$ is in the triangle. The following proposed algorithms for randomly sampling a triangle each come up with values for the coefficients that define p.

Algorithm 1: Choose three random values $r1$, $r2$, and $r3$. Set $\alpha = \frac{r1}{r1+r2+r3}$, $\beta = \frac{r2}{r1+r2+r3}$, and $\gamma = \frac{r3}{r1+r2+r3}$.

Algorithm 2: Choose two random values $r1$ and $r2$; but if $r1 + r2 > 1$, then replace $r1$ by $1 - r1$ and $r2$ by $1 - r2$. Set $\alpha = 1 - r1 - r2$, $\beta = r1$ and $\gamma = r2$.

Algorithm 3: Choose two random values $r1$ and $r2$. Set $\alpha = 1 - \sqrt{r1}$, $\beta = \sqrt{r1} * r2$ and $\gamma = \sqrt{r1} * (1 - r2)$.

Write a program which accepts the vertices of a triangle, and a point count N, computes N random points in the triangle according to each of the 3 algorithms, and makes a separate plot of the sample points for each algorithm.

Test your code with N=1000 points on the triangle {(4,0),(3,4),(0,1)}.

Turn in your three plots. I expect that you will see one algorithm behaves differently from the other two. You do not have to explain this difference, but please comment on it, and describe what you see.

Problem 2: *Triangle Quadrature*

Let T be the triangle whose vertices {A,B,C} are {(0,1),(3,0),(2,5)}. The integral of a function $f(x, y)$ over the triangle can be approximated by evaluating the function at N points selected uniformly at random from T, averaging these values, and multiplying by the area of T.

Estimate the integral of the function $f(x) = x^2$ over T using N = 10, then 100, then 1000, then 10000, then 100000, and:

1. Algorithm 1;
2. Algorithm 2;
3. Algorithm 3.

Turn in your 15 computed estimates for the integral, and make any comments you can think of about the fact that the exact integral is 46.25.

You are not required to do problems 3 and 4 since we only have one lab session!

Problem 3: *Triangle Properties*

Let T be the triangle whose vertices {A,B,C} are {(0,1),(3,0),(2,5)}. Compute the following quantities:

1. the angles at vertices A, B and C;
2. the area;
3. the centroid;
4. the distance from the point (7.2,4.0) to T.

Turn in your numbered answers to these questions.

Problem 4: *Polygon Triangulation*

Get a copy of the data describing the *i18* polygon.

Get a copy of the *triangulate.m* program.

Using the *triangulate* program, determine a triangulation of the *i18* polygon. This will be a list of triples of nodes that form triangles. Using this triangulation, answer the following questions:

1. what is the area of the polygon?
2. what is the centroid ("center of mass") of the polygon?
3. what is the distance of the point (-5,12) to the polygon?

Turn in your numbered answers to these questions.

Data: *available on Blackboard:*

- *i18_nodes.txt* lists the nodes for the i18 polygon.
- *triangulate.m* computes the triangulation of a polygon.

These files will also be available by going to the web page
http://people.sc.fsu.edu/~jburkardt/latex/asa_2011_geometry_lab/asa_2011_geometry_lab.html

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