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1 Malaria

"Malaria is a serious and sometimes fatal disease caused by a parasite that commonly infects a certain type of mosquito [that] feeds on humans" (CDC). Last year, there were 214 million Malaria cases and 438,000 Malaria deaths (WHO). Malaria is both preventable and curable, however, and Malaria death rates among at-risk populations have fallen by 60% in the last 15 years (WHO).

Given that Malaria is preventable, curable, and very harmful, it's important to determine the most effective measures to combat the illness. A mathematical model of Malaria infection that includes factors like mosquito population size and mosquito bite rate will enable us to understand what drives Malaria incidence. This information can be used by public health professionals to further alleviate the effect of Malaria.

2 Model Design

The details of the model for Malaria incidence were provided in "Mathematical Modeling for Malaria Control.pdf". Basically, the model assumes no latent period for infection, the possibility to become immune, and therefore three populations of human villagers: sick, immune, and healthy. Mosquitos are not treated as capable of becoming immune, and thus there are two populations of mosquitos: infected and healthy. The size of each of these populations is calculated at each time step in our program (attached).

The size of each population is a function of other populations' sizes, and several parameters that define quantities such as the mosquito bite rate and the human birth rate. See section "4.1 Sensitivity Analysis" below for a discussion and graphical depictions of the impact of varying some of these parameters.

The model was converted into a Python program that was provided to us in an almost-complete state. Our role was developing functions that would define the sizes of several populations at each time step. Importantly, the definition for the size of the population of healthy villagers was provided to us, and we made changes to it for a couple of reasons: 1) healthy villager population size was not a function of the sizes of the immune and sick villager populations, even though these two groups could be giving birth to healthy villagers; 2) healthy villager population size should never be lower than 0.

These two reasons for changing the provided code for healthy villagers also influenced our definitions of the other populations. Basically, we disallowed negative population values, and we assumed that infected organisms (mosquitos and humans) would give birth to healthy organisms–in line with the literature on the topic (Opare, 2010).

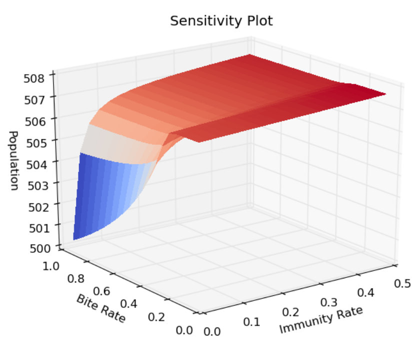
3 Model Solution

This model was solved using a time-stepping method to solve the system of coupled equations that made up the model, very similar to the Forward Euler method. This method is an explicit method for solving models such as this in the sense that each can be directly solved using information from the previous timesteps and does not require solving a nonlinear system with dependence on the current time at each timestep. For implementation, we used Python with packages MatPlotLib and Numpy for plotting and arrays. Numpy proved most useful in the sensitivity analysis as it was required to compute the grid for the 3D plots shown in the next section.

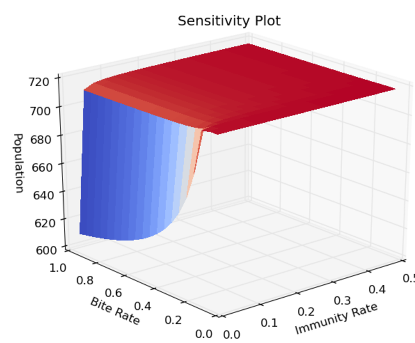
4 Results and Conclusions

4.1 Sensitivity Analysis

The immunity and the bite rate are the two parameters which were analyzed for sensitivity using total population as a sensitivity metric. As the plot below shows, the immunity rate plays a major role in final population. Bite rate is only able to cause a population crash in the event that the immunity rate is sufficiently small, usually less than 10%. The simulation was also run out to 5000 days (nearly 14 years) in order to see how the populations would respond to varying parameters in the long term. This is the subject of the next figure. As you can see, the population will still start to crash, as expected. Though oddly the effect of bite rate, on the long-term, is not felt until the immunity rate gets very low. This may be an effect of the underlying model or the assumptions made in the model.



(a) 200 Day Short-Term Sensitivity



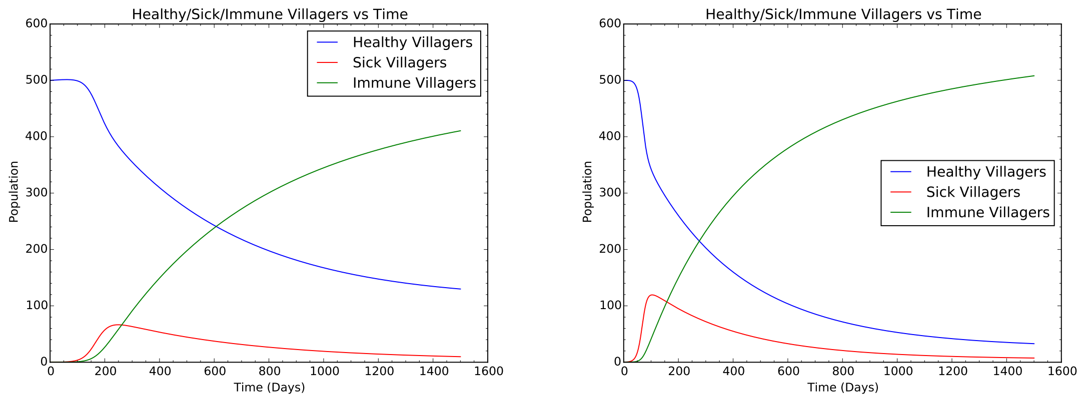
(b) 5000 Day Long-Term Sensitivity

5 Model Additions

5.1 Relapse

A major assumption of our model is that all healthy people in our population have the same probability to become sick for all time. With many diseases, once an individual has been infected and has recovered they are more likely to relapse in to a sick state. In order to incorporate the effects of relapse into our model we had to define a new population of villagers who previously had the disease and recovered. The recovered villagers would be a subset of the healthy villagers and would lead to an increase in the growth of the sick villager population.

$$Relapse = RecoveredVillagers * RelapseRate$$
$$RecoveredVillagers = Recovered - Death - Relapse - Infected$$



(a) No relapse

(b) Relapse

Figure 2: The introduction of relapse results in a larger initial sickness spike but also a faster immunity saturation.

A relapse rate of 17% was provided and was taken to mean that after each day there was a 17% chance for previously sick villagers to become sick again. Modification to the healthy villagers and sick villagers were made such that

$$SickVillagers = Infected - Recover - Immune - SickDies + Relapse$$
$$HealthyVillagers = Births + Recover - HealthyDies - Infected - Relapse$$

allowing for the relapsed villagers to be removed from the healthy group and added to the sick population. Immune villager population was not altered in this modification.

We see that for the case of no relapse around the 200 day mark there there is a rapid increase in the sick population and leading to a decrease in the healthy population as well as an increase in the immune population. For the introduction of a relapse model we see that this spike in sickness occurs sooner, around the 100 day mark, and is almost twice the size in magnitude.

An interesting effect was also observed with respect to the immune villager population. The only way for a villager to become immune in our current model is to become sick in a previous time step. We observed that the introduction of relapse into our model increases the number of sick villagers which in turn results in an increase in the number of immune villagers. As long as the death rate for a sick villager is not to high, the introduction of relapse seems increase the rate at which the village is saturated with an immune population.

5.2 Seasonal Mosquito Population

According to the seasonal increase in the number of mosquitoes, such as a rainy season in tropical areas, we have to figure out the new rates for mosquitoes birth and death. The mosquito birth rate "mbr" and death rate "mdr" are then defined by:

$$mbr = \frac{1}{2}[cos(2\pi t/365 + 1) * MaxMosquitosBirthRate]$$

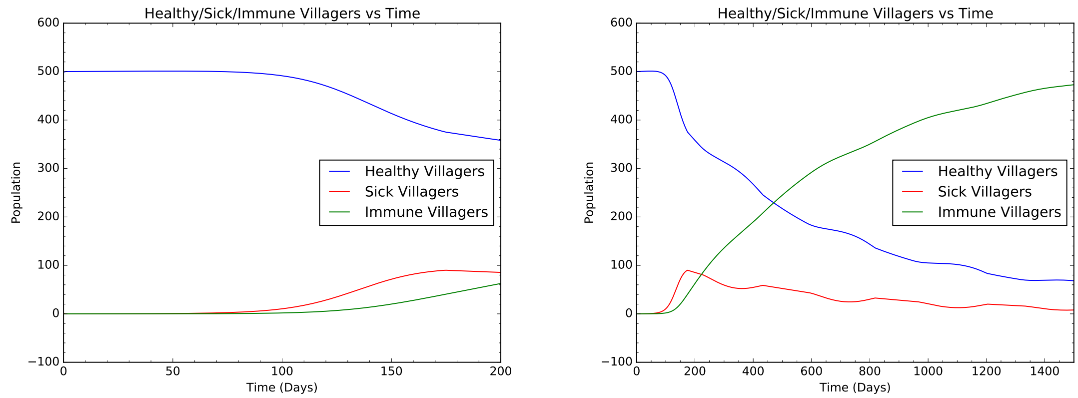
$$mdr = \frac{1}{2}[cos(2\pi t/365 + 1) * MaxMosquitosDeathRate]$$

Where :

$$MaxMosquitosBirthRate = 0.02$$

$$MaxMosquitosDeathRate = 0.022$$

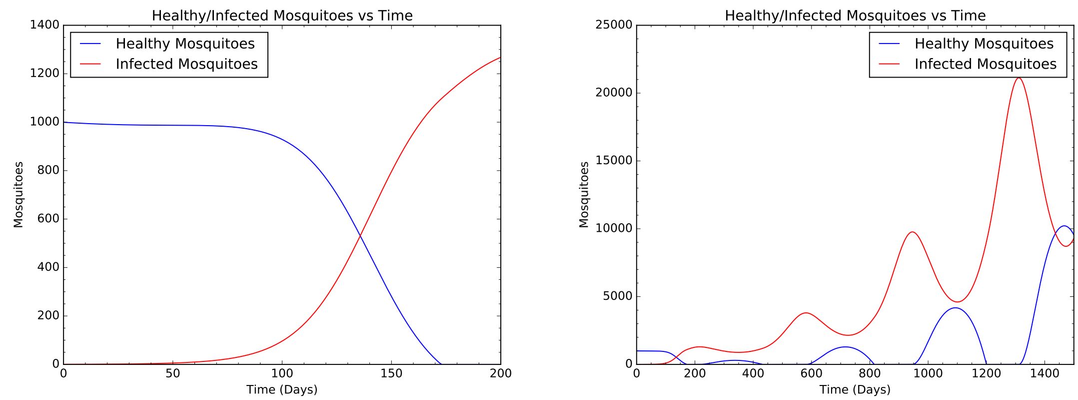
The introduction of a seasonal dependence on the mosquitoes birth and death rates was implemented and tested for 200 and 1500 days.



(a) 200 Days (b) 1500 Days

Figure 3: The introduction of seasonal change increases the rate at which the healthy population decreases.

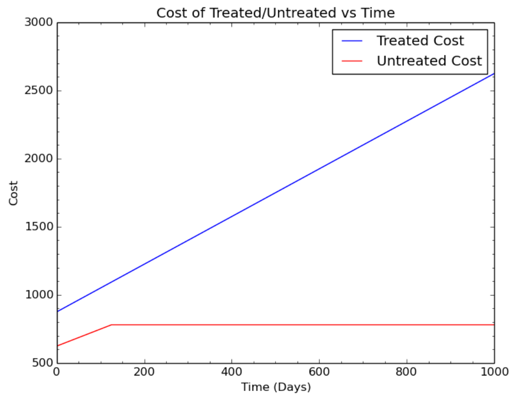
For the 200 day scenario only the beginning of the seasonal effects can be seen. For longer run times the oscillator nature of the mosquito population is much more dramatic. As you can see, the number of healthy villagers decreases over time like the original model but variation in mosquito population lead to a less smooth transition.



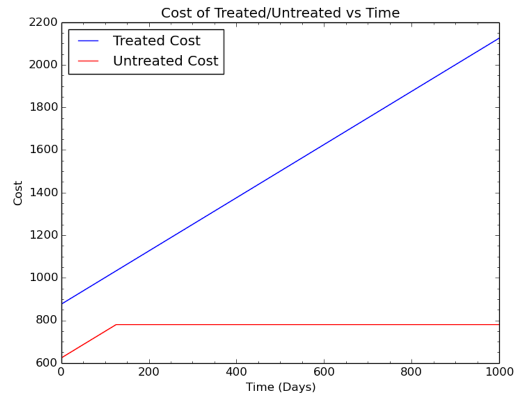
(a) 200 Days (b) 1500 Days

Figure 4: The introduction of seasonal change increases the rate at which the healthy mosquitoes decreases.

5.3 Preventative Net Cost Analysis



(a) For the treated graph, treated nets were bought each time there were new villagers and untreated nets were bought for the untreated graph



(b) For the treated graph, untreated nets were bought each time there were new villagers and untreated nets were bought for the untreated graph

Introduction of nets into a village is a common tool to fight against the spread of malaria. A cost analysis of using such nets was performed taking into account two possible net types. The first was a simple net with a cost of $2.50 and the second was a treated net with a cost of $3.50. Insecticide treated nets (ITN) reduced the incidence of uncomplicated malarial episodes by 50% compared to no nets, and 39% compared to untreated nets. For this analysis it was assumed that one net could cover two people

The difference between the cost where all treated nets and all untreated nets were bought was 1845.0. The difference in the total healthy villagers was 296.72. Keeping everything the same, but making it so each new net bought is an untreated one we get a cost difference of 1345.0 and healthy villager difference of 296.64. The cost difference between of buying all treated nets and half treated, half untreated is 1005.0 and the healthy villager difference is 178.25. It is clear that buying the treated net for each new villager is not worth the much higher cost in the long term.

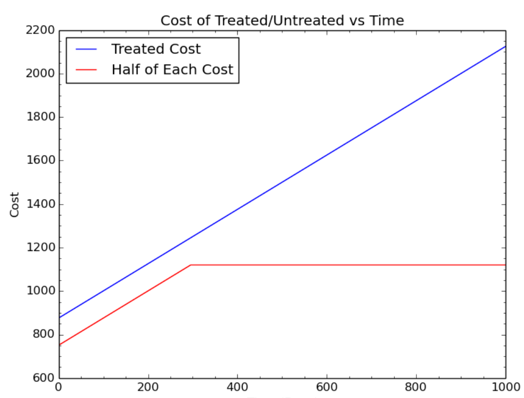


Figure 6: For the treated graph, treated nets were bought each time there were new villagers and for the half graph, originally half the nets were treated and half untreated and then u ntreated nets

On a short time scale, buying the villagers the more expensive net will stop the population from decreasing immediately, but even with all untreated nets, the population will come back and increase later. I would say that originally buying a little more (maybe 60-70 percent) treated nets and then buying only untreated is the most cost efficient way of buying the nets to control the decrease of the population.

6 References

CDC. www.cdc.gov/malaria/about/faqs.html

WHO. http://www.who.int/mediacentre/factsheets/fs094/en/

Opare, 2010. ”Congenital Malaria in Newborn Twins”.