Ancient Astronomy

Lectures 5-6

Course website**:** www.scs.fsu.edu/~dduke/lectures

Lectures 5-6

- *Almagest* Books 9–13
- geocentric vs. heliocentric point of view
- the wandering stars, or planets
- the two anomalies
- the eccentric plus epicycle and its problems
- the equant
- latitude
- distances
- the background

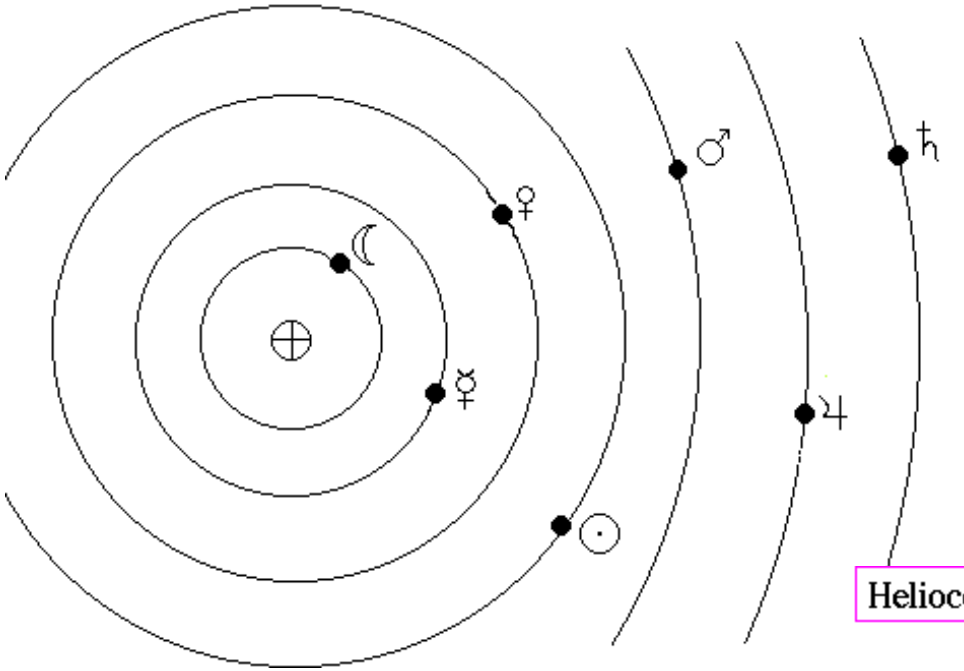
In reality the Earth and all the other planets revolve around the Sun.

Nevertheless, we can imagine a reference frame in which the Earth is at rest, and ask “what would a correct theory look like in that reference frame?”

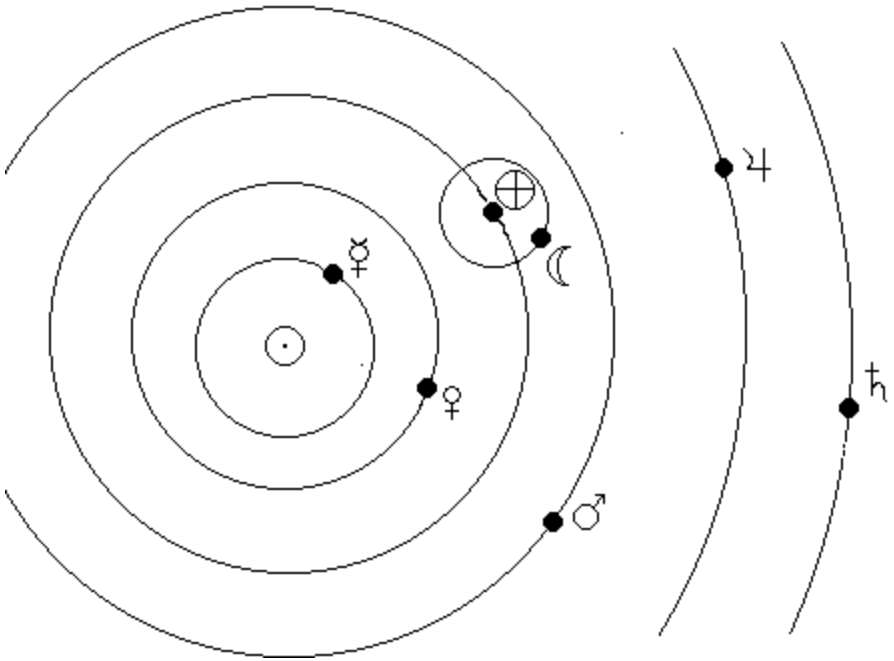
Answer: it would look *very much* like the theory created by the Greek astronomers.

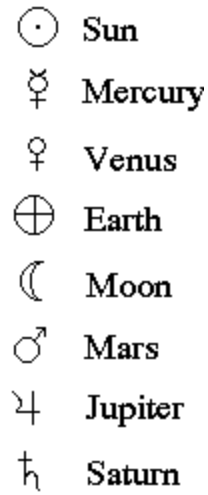
And note: modern astronomers compute first the planets orbiting the Sun, and then have to figure out the position of the planet relative to the Earth.

Geocentric Solar System Model



Heliocentric Solar System Model





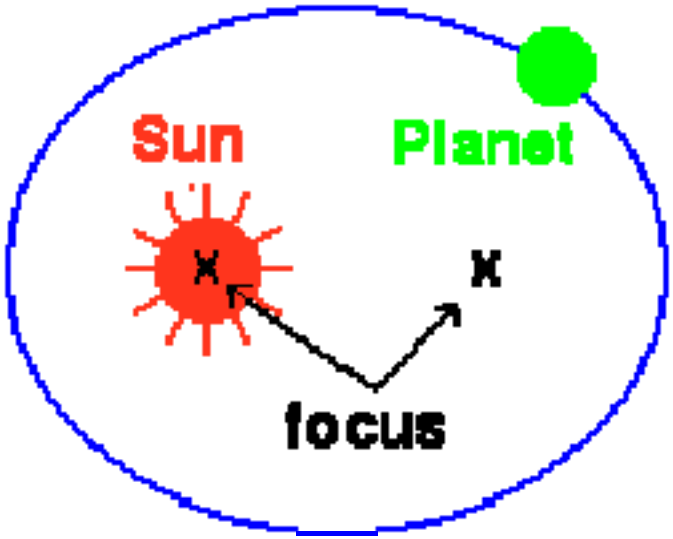
Problems for heliocentric theory:

- Earth in motion??? can't feel it
- no parallax seen in stars

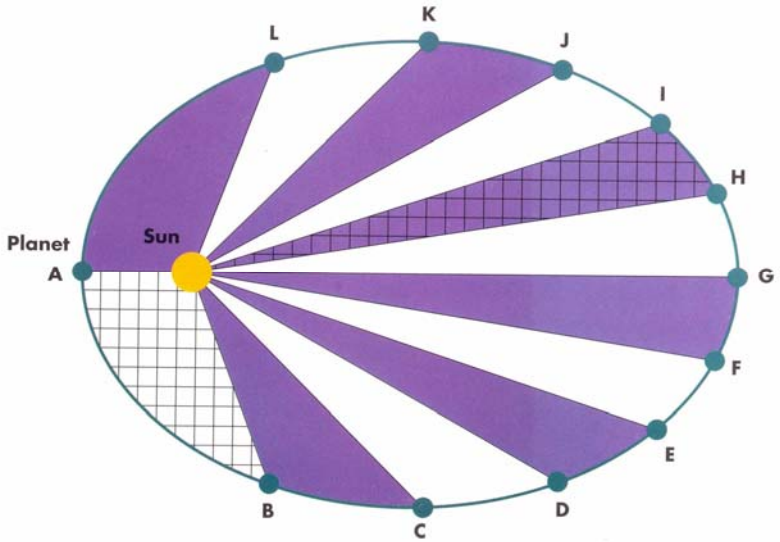
geocentric = ego-centric = more "natural"

Kepler's Three Laws of planetary motion:

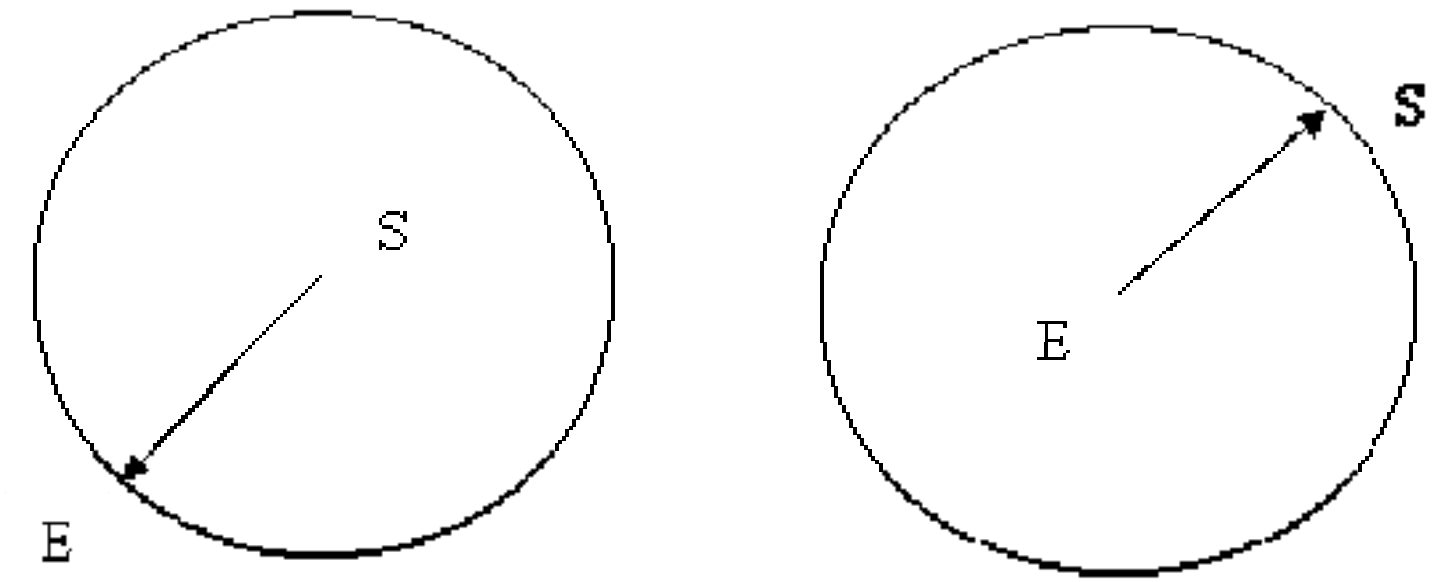
1. orbits are ellipses, Sun at focus



2. equal area in equal time

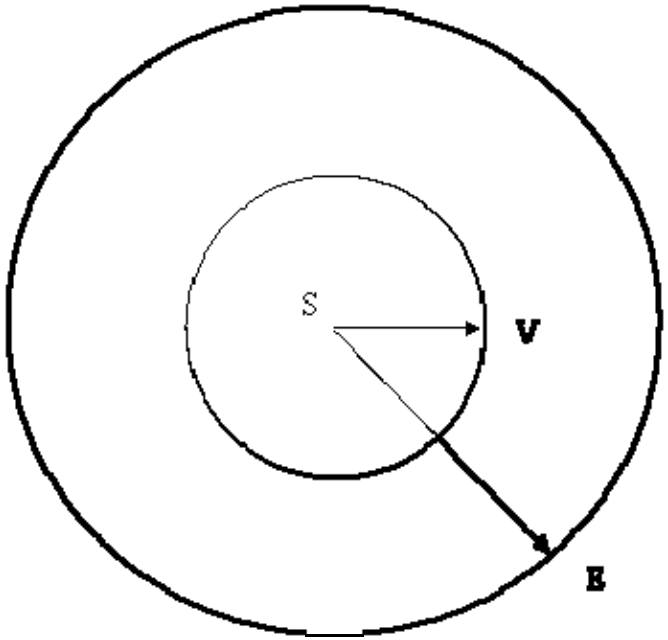


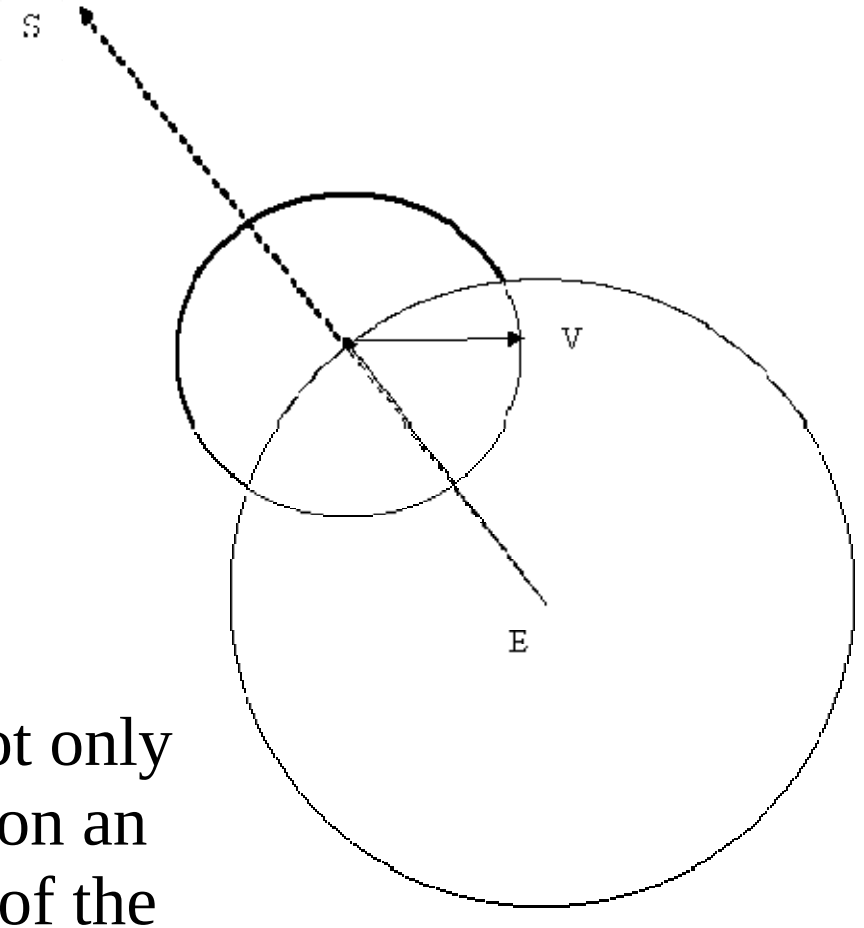
3. $\frac{P^2}{a^3} = 1$ for each planet



Instead of the Earth circling the Sun, we would have the Sun circling the Earth.

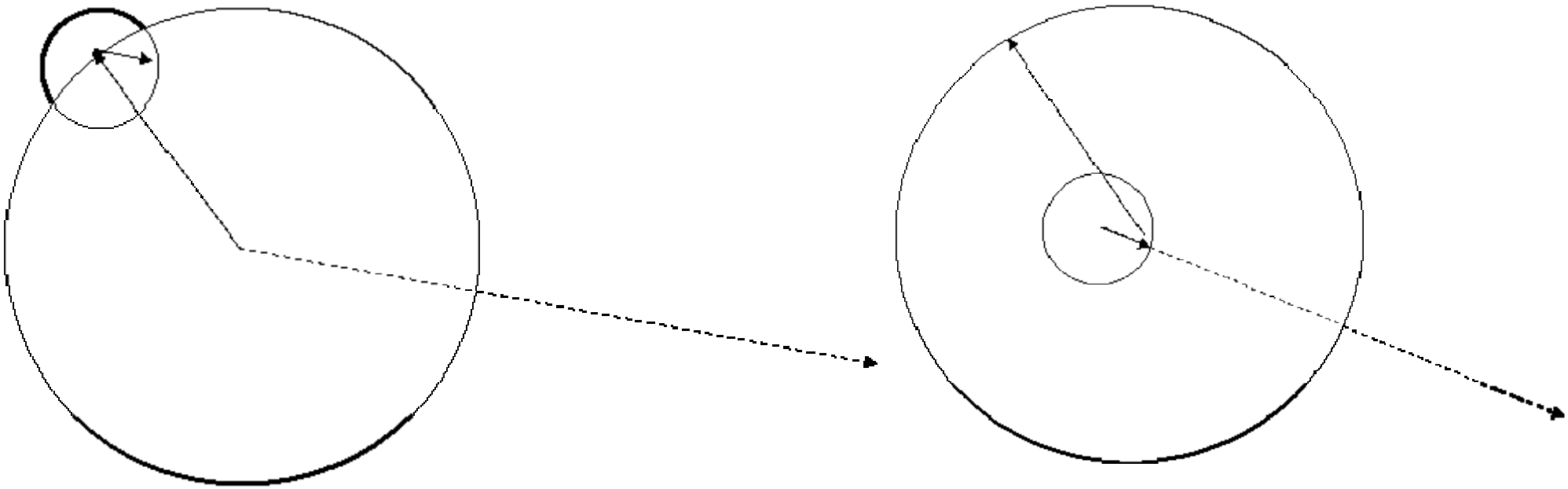
http://www.csit.fsu.edu/~dduke/venhelio.html





For the inner planets, not only does the planet revolve on an epicycle, but the center of the epicycle is always lined up with the Sun.

For the outer planets, the radius of the epicycle is always parallel to the direction of the Sun from the earth.



http://www.csit.fsu.edu/~dduke/juphelio.html

All of the planets have, from time to time, a *retrograde* motion, i.e. the slow motion from west to east stops, then reverses into an easy to west motion, then stops again and resumes a west to east motion.

http://www.astronomynotes.com/nakedeye/animations/retrograde-anim.htm

In reality this happens because planets closer to the Sun move faster than planets farther from the Sun.

In a geocentric theory, this happens because of the counter-clockwise motion on the epicycle.
http://www.astro.utoronto.ca/~zhu/ast210/both.html
http://www.scs.fsu.edu/~dduke/models.htm

And note the close relation to the Sun. In a heliocentric picture it is clear that in retrograde the Sun–Earth–planet are in a line. In a geocentric picture it is not required, but the Greeks knew that had to assume it to be true.

Counting the number of retrograde episodes and planetary orbits over many years gives the period relations:

planet	years	orbits	retrogrades
Saturn	59	2	57
Jupiter	71	6	65
Mars	79	42	37
Venus	8	13	5
Mercury	46	191	145

Note that the solar year is somehow involved for every planet! Such relations are completely *ad hoc* in a geocentric view but exactly as expected in a heliocentric view.

In the *Almagest* Ptolemy says little about the *distances* to the planets:

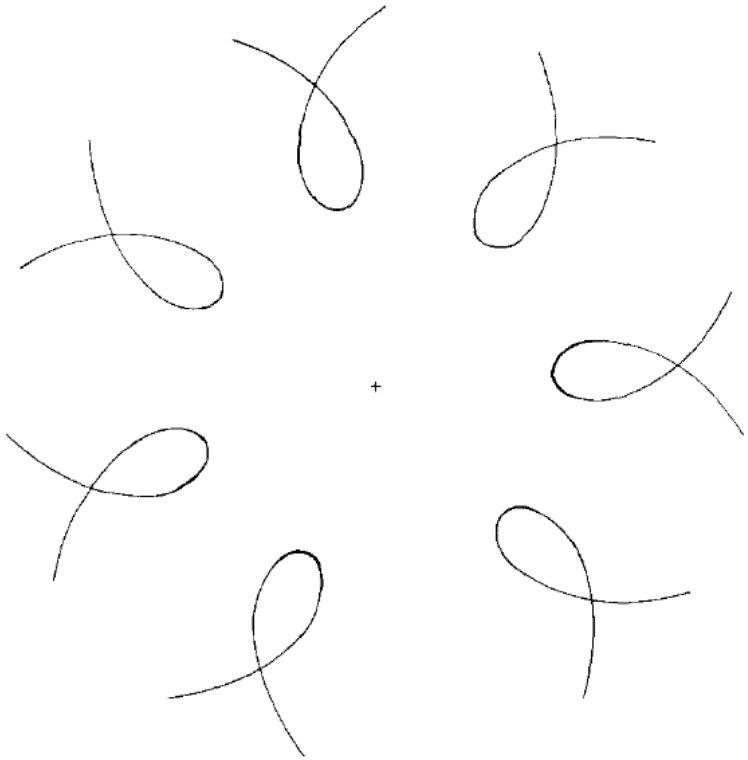
First, then, [to discuss] the order of their spheres, which are all situated [with their poles] nearly coinciding with the poles of the inclined, ecliptic circle: we see that almost all the foremost astronomers agree that all the spheres are closer to the earth than that of the fixed stars, and farther from the earth than that of the moon, and that those of the three [outer planets] are farther from the earth than those of the other [two] and the sun, Saturn's being greatest, Jupiter's the next in order towards the earth, and Mars' below that. But concerning the spheres of Venus and Mercury, we see that they are placed below the sun's by the more ancient astronomers, but by some of their successors these too are placed above [the sun's],[1] for the reason that the sun has never been obscured by them [Venus and Mercury] either. To us, however, such a criterion seems to have an element of uncertainty, since it is possible that some planets might indeed be below the sun, but nevertheless not always be in one of the planes through the sun and our viewpoint, but in another [plane], and hence might not be seen passing in front of it, just as in the case of the moon, when it passes below [the sun] at conjunction, no obscuration results in most cases.[2]

For all the models Ptolemy assumes a deferent circle of radius $R = 60$ and an epicycle of radius $r < 60$. Comparing the heliocentric distances and the *Almagest* geocentric distances gives

	modern		*Almagest*
planet	*a*	*r*	*r*
Mercury	0.3871	23;14	22;30
Venus	0.7233	43;24	43;10
Mars	1.5237	39;22	39;30
Jupiter	5.2028	11;32	11;30
Saturn	9.5388	6;17	6;30

As far as we know, nobody after Aristarchus (*ca.* 230 B.C.) and before Copernicus (A.D. 1540) was willing to make the leap to the heliocentric picture.

Like the Sun and Moon, the speed of the planets also varies smoothly as they circle the zodiac, so the planetary orbits each have an apogee and a perigee.



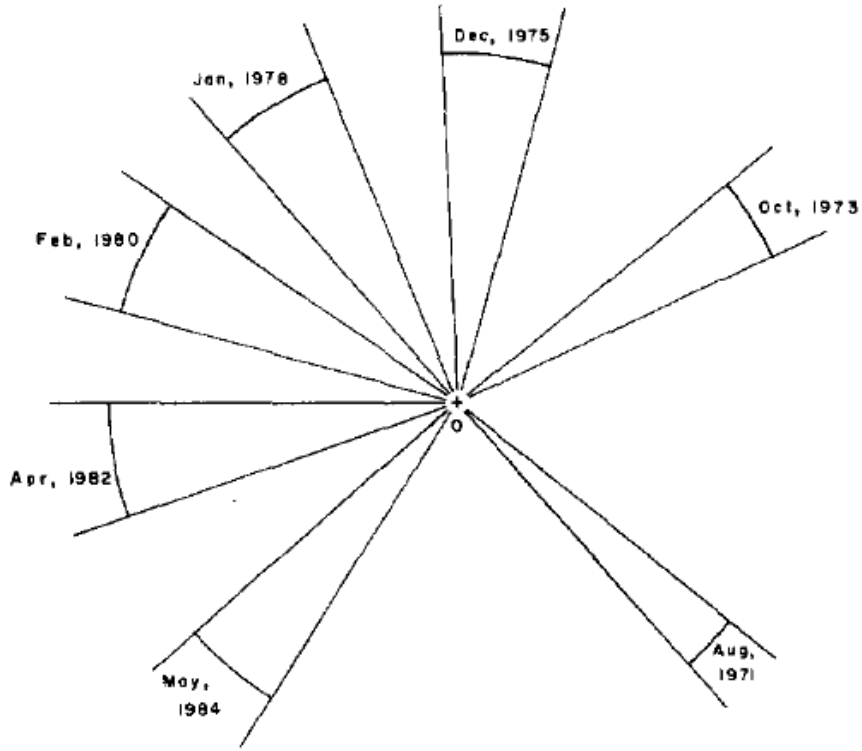
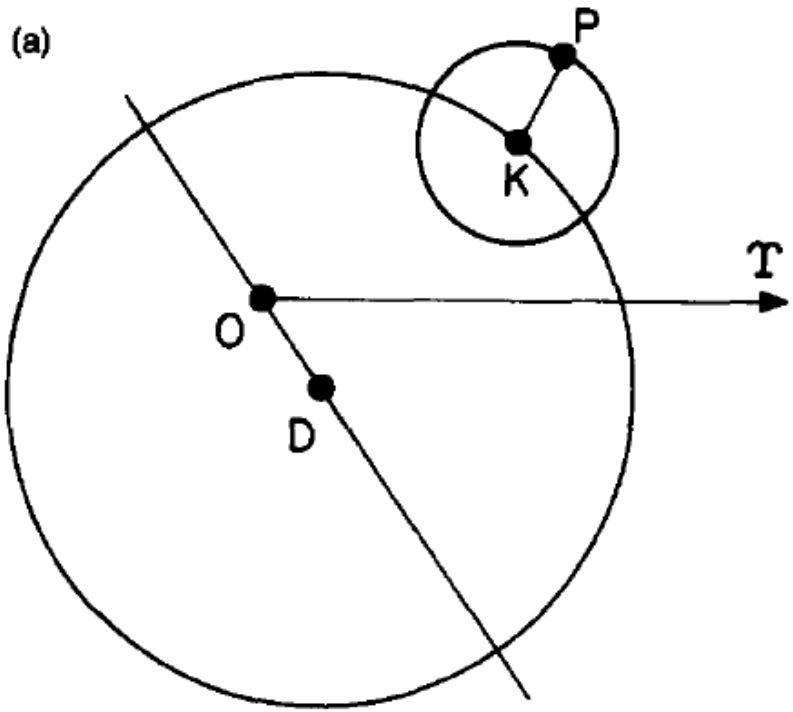
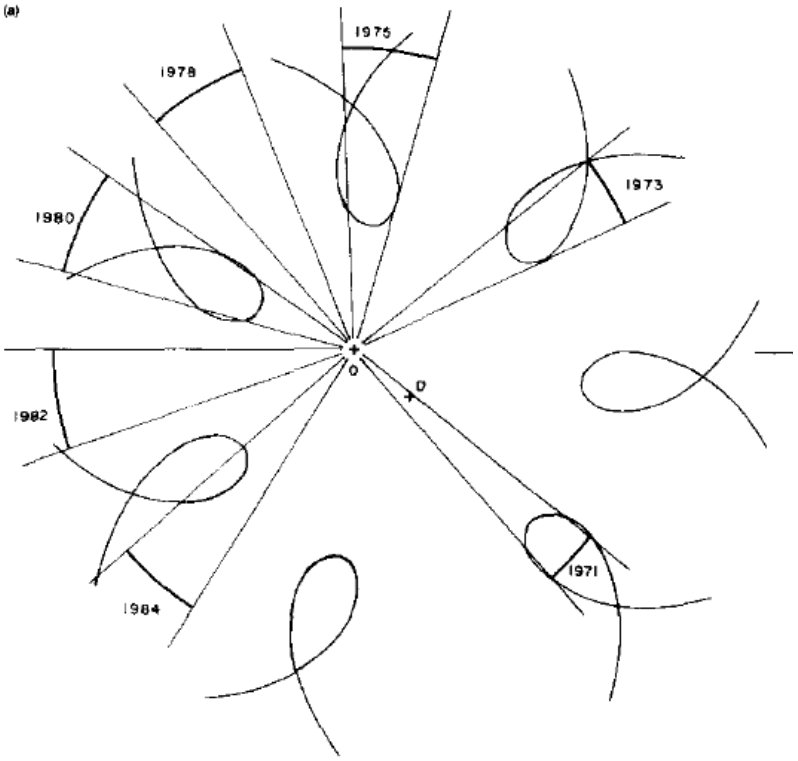
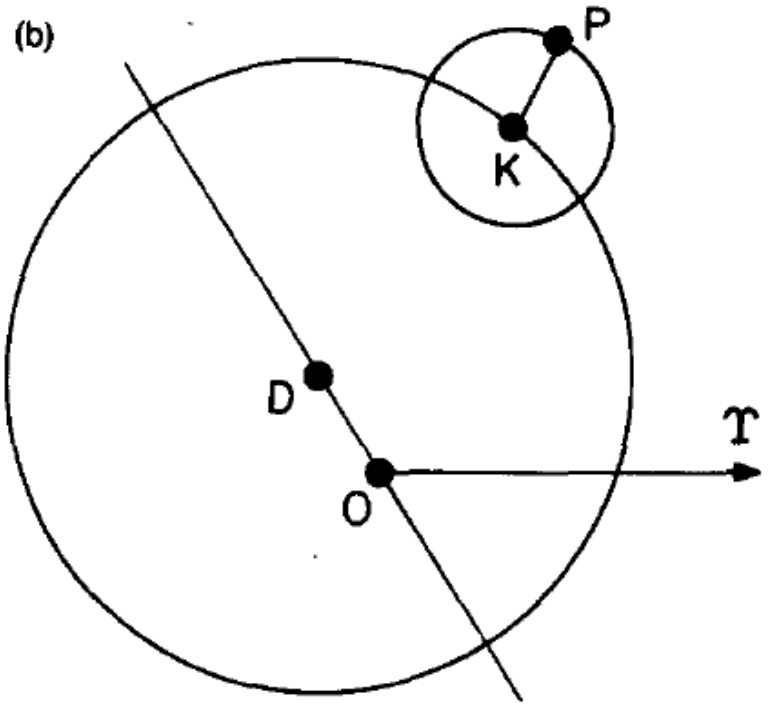
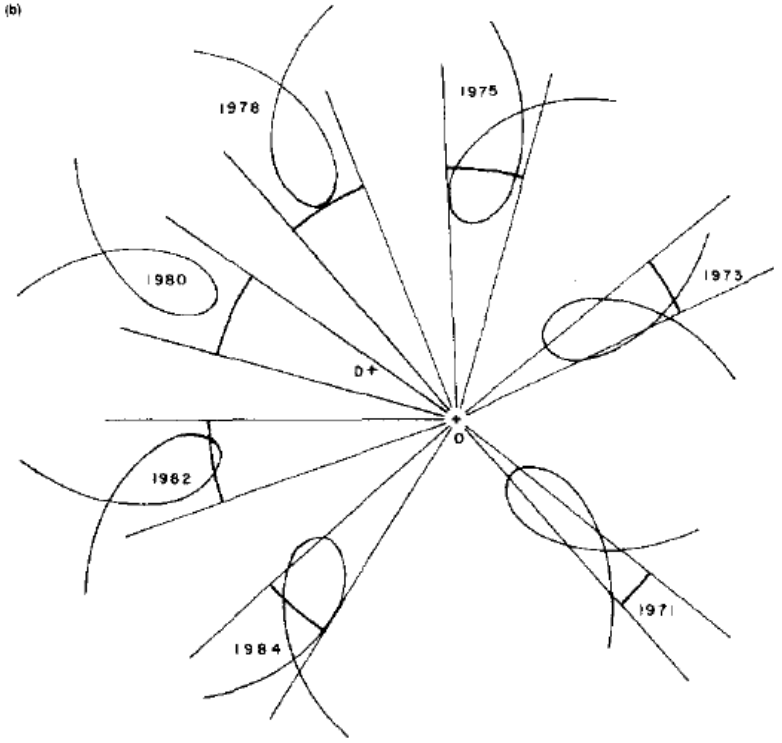


Fig. 4. Retrograde loops of Mars generated by the zero-eccentricity model of Fig. 5.


(a)
P
K
O
D
ϒ


(a)
1975
1978
1973
1980
1982
1971
1984


(b)
P
K
D
O
ϒ


(b)
1978
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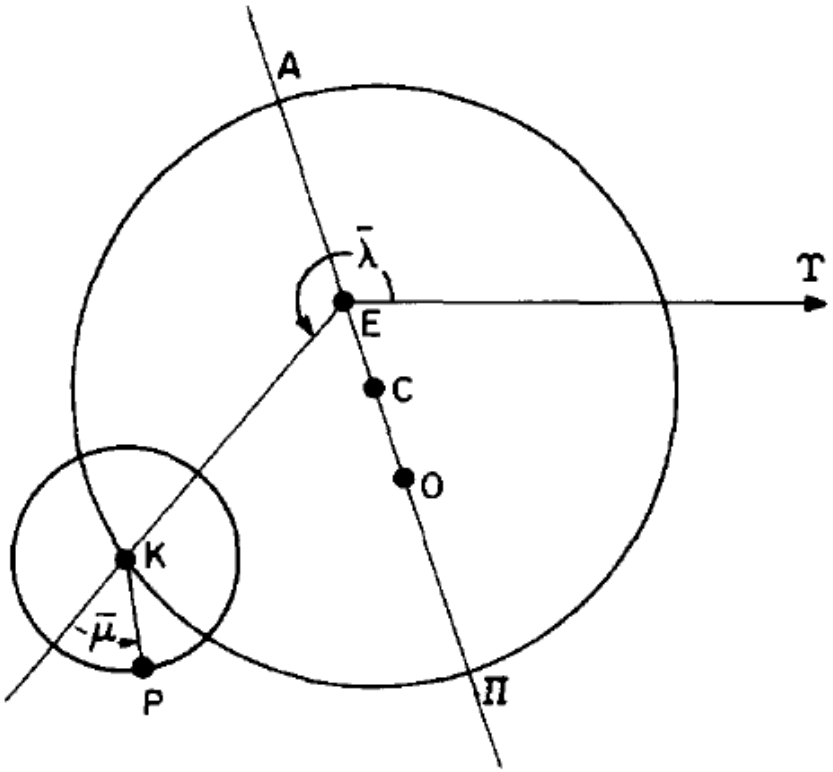


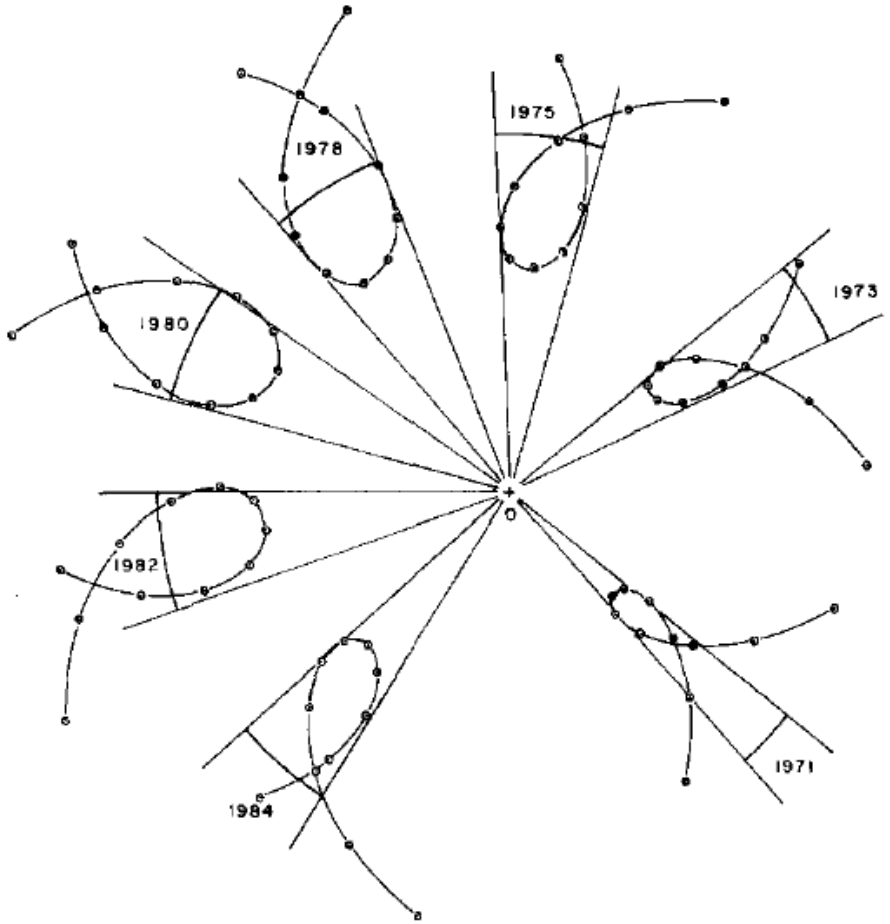
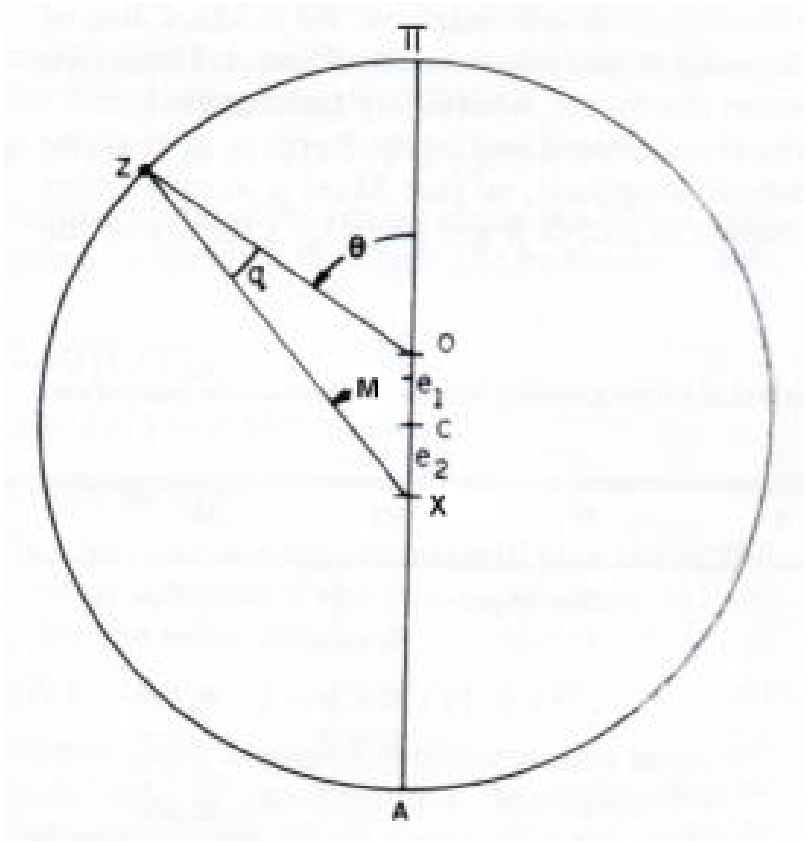
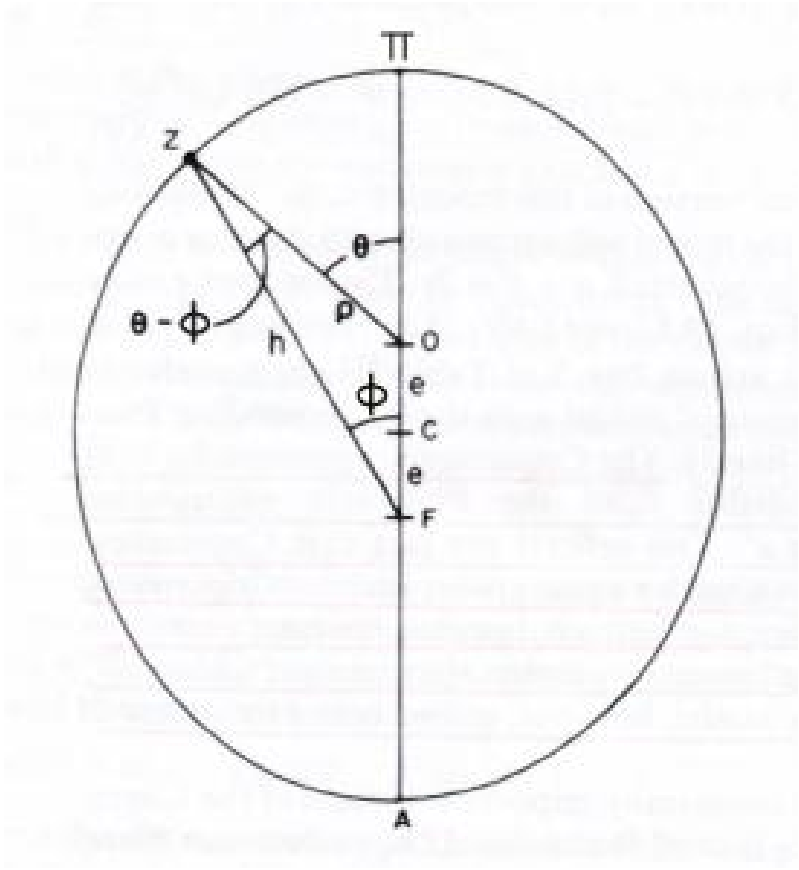


Fig. 1. Ptolemy's theory of longitudes for Venus, Mars, Jupiter, an urn.

A new idea, the equant, solves the problem.





The equant is very similar to Kepler's ellipse, and accounts very well for Kepler's 1st and 2nd Laws.

www.scs.fsu.edu/~dduke/kepler.html

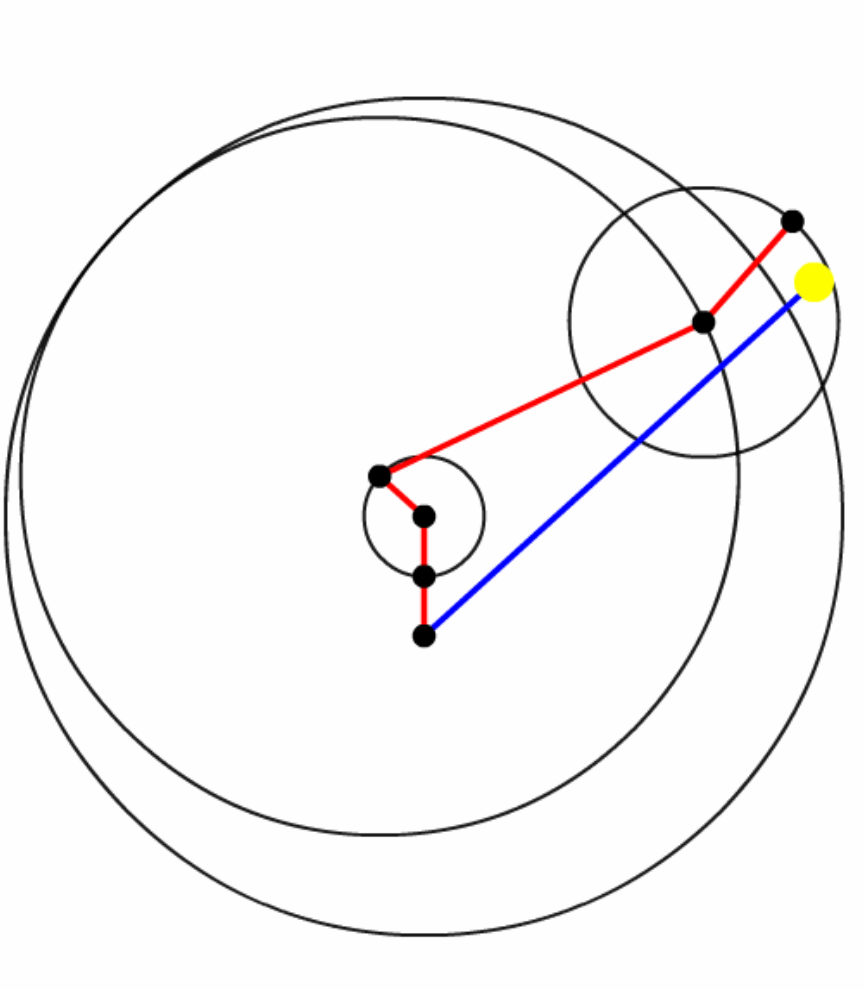
http://people.scs.fsu.edu/~dduke/kepler3.html

Combining the periods and distances gives Kepler's 3rd Law:

	a	a^3	Period	P^2	P^2/a^3
Mercury	0.38	0.05	0.24	0.06	1.10
Venus	0.72	0.37	0.62	0.38	1.02
Earth	1.00	1.00	1.00	1.00	1.00
Mars	1.52	3.50	1.88	3.54	1.01
Jupiter	5.22	142.02	11.83	140.03	0.99
Saturn	9.23	786.53	29.50	870.25	1.11

So the *Almagest* models are indeed *very much* like the real planetary orbits when viewed from Earth.

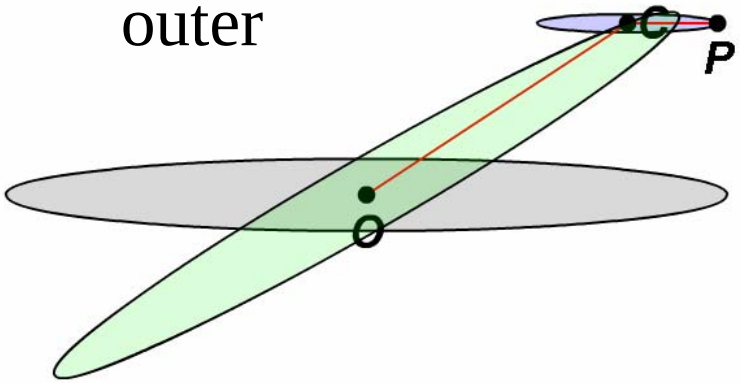
For some reason Ptolemy makes the model for Mercury more complicated.



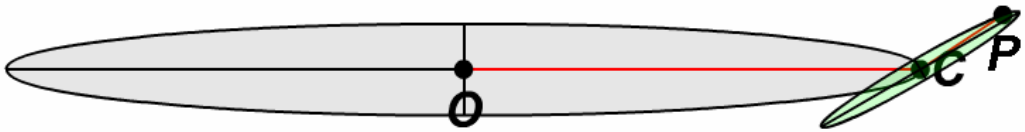
http://people.scs.fsu.edu/~dduke/mercury.html

Like the Moon, the planet orbits are tilted relative to the Sun's orbit.

outer



inner



Outer planet http://people.scs.fsu.edu/~dduke/latitude.html
Inner planet http://people.scs.fsu.edu/~dduke/latitude2.html
Note that these make good sense in a heliocentric view.

In the *Planetary Hypotheses* Ptolemy writes:

> If (the universe is constructed) according to our description of it, there is no space between the greatest and least distances (of adjacent spheres), and the sizes of the surfaces that separate one sphere from another do not differ from the amounts we mentioned. This arrangement is most plausible, for it is not conceivable that there be in Nature a vacuum, or any meaningless and useless thing. The distances of the spheres that we have mentioned are in agreement with our hypotheses. But if there is space or emptiness between the (spheres), then it is clear that the distances cannot be smaller, at any rate, than those mentioned.

http://people.scs.fsu.edu/~dduke/ptolemy.html

Using his “nesting” assumption Ptolemy gets:

Planet	Mean Distance in Earth Radii	Apparent Diameter compared to the Sun's	True Diameter compared to the Earth's	Volume compared to the Earth's
Moon	48	1⅓	¼ + 1/24	1/40
Mercury	115	1/15	1/27	1/19,683
Venus	622½	1/10	¼ + 1/20	1/44
Sun	1,210	1	5½	166⅔
Mars	5,040	1/20	1⅐	1½
Jupiter	11,504	1/12	4⅓ + 1/40	82¼ + 1/20
Saturn	17,026	1/18	4¼ + 1/20	79½
1. Magn. Stars	20,000	1/20	4½ + 1/20	94⅙ + ⅛

Early Greek Planetary Theories

The Keskintos Inscription (found on Rhodes about 1890) and probably carved about 100 B.C.

	i	ii	iii	iv	v	vi	vii	viii
	Mercury	[In relative position]	[passages]	xxxx	Mercury	[In] relative position	passages	[91]84xx
	Mars	In longitude	zodiacals	15492	Mars	In longitude	zodiacals	154920
	Mars	In latitude	tropicals	15436	Mars	In latitude	tropicals	154360
	Mars	In depth	revolutions	4096x	Mars	In depth	revolutions	401650
5	Mars	In relative position	passages	13648	Mars	In relative position	passages	136480
	Jupiter	In longitude	zodiacals	2450	Jupiter	In longitude	zodiacals	24500
	Jupiter	In latitude	tropicals	2456	Jupiter	In latitude	tropicals	24560
	Jupiter	In depth	revolutions	24260	Jupiter	In depth	revolutions	242600
	Jupiter	In relative position	passages	26690	Jupiter	In relative position	passages	266900
10	Saturn	In longitude	zodiacals	992	Saturn	In longitude	zodiacals	9920
	[Saturn]	In latitude	tropicals	989 216	Saturn	In latitude	tropicals	9896
	[Saturn]	In depth	revolutions	27176	Saturn	In depth	revolutions	271760
	[Saturn]	[In] relative position	passages	28148	Saturn	In relative position	passages	281480

]… A circle comprises 360 degrees or 9720 *stigmai*. A degree comprises 2[7] points.

15] to … a thank-offering.

These are period relations as before, but much longer:

planet	years	orbits	retrogrades
Saturn	29140	992	28148
Jupiter	29140	2450	26690
Mars	29140	15492	13648
Venus	?	?	?
Mercury	?	?	?

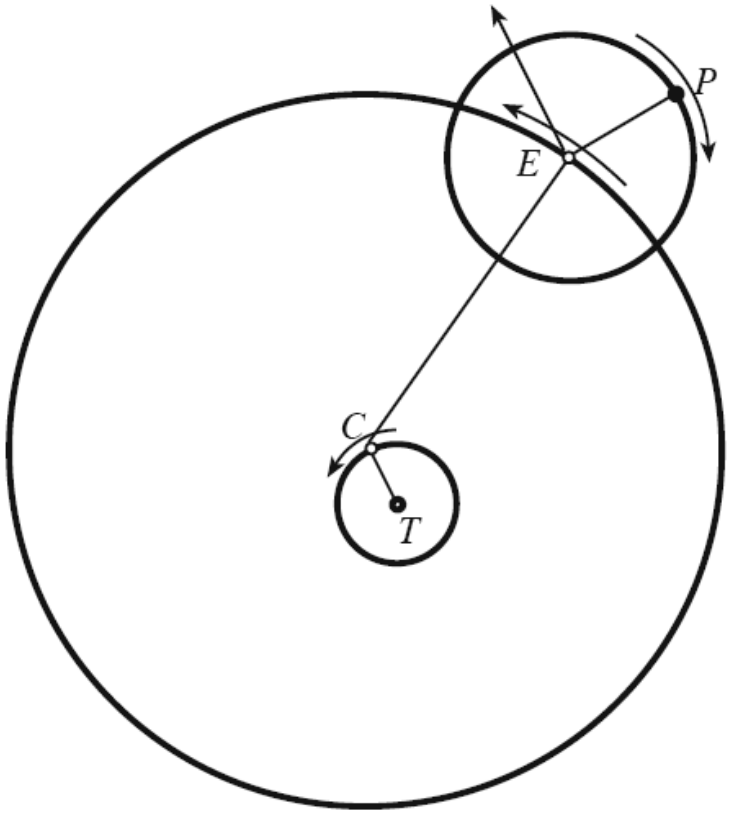
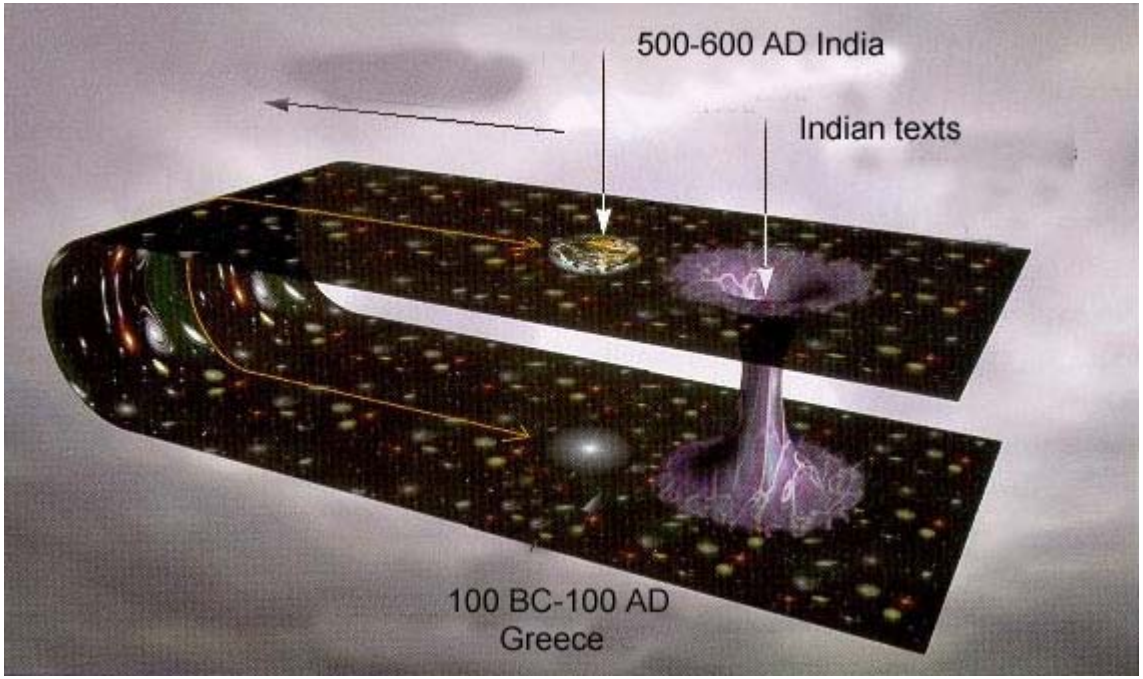


Fig. 2. Possible epicyclic model for Jupiter or Saturn in the Keskintos Inscription.

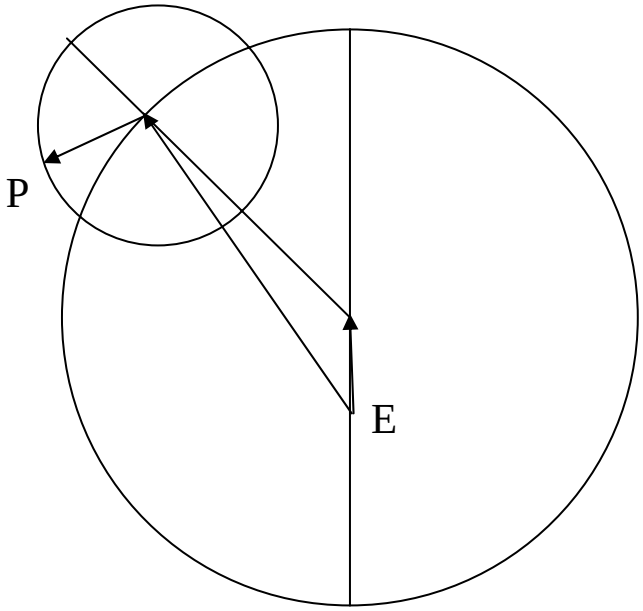
The texts of ancient Indian astronomy give us a sort of wormhole through space-time back into an otherwise inaccessible era of Greco-Roman developments in astronomy.

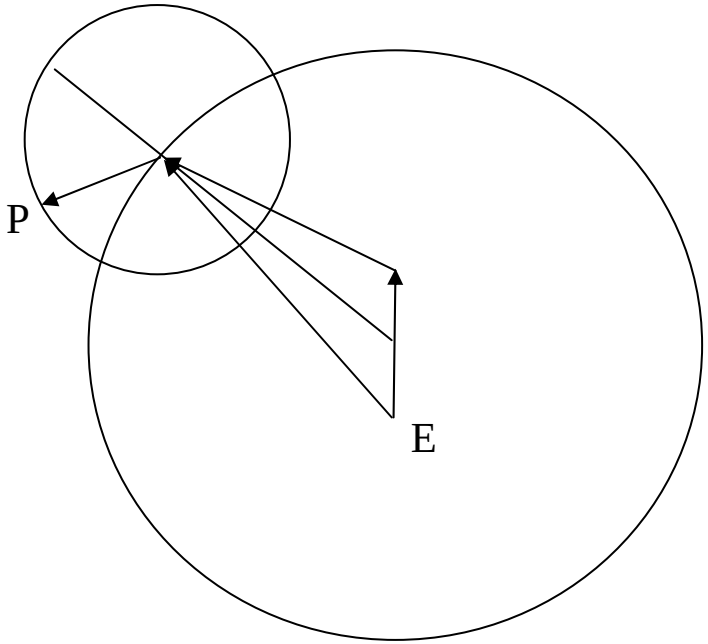


Indian Planetary Theories

Conventional wisdom:

"The orbits of the planets are concentric with the center of the earth. The single inequalities recognized in the cases of the two luminaries are explained by *manda*-epicycle (corresponding functionally to the Ptolemaic eccentricity of the Sun and lunar epicycle, respectively), the two inequalities recognized in the case of the five star-planets by a *manda*-epicycle (corresponding to the Ptolemaic eccentricity) and a *sighra*-epicycle (corresponding to the Ptolemaic epicycle). **The further refinements of the Ptolemaic models are unknown to the Indian astronomers.**"





The Indian theories have even longer period relations:

planet	years	orbits	retrogrades
Saturn	4,320,000	146,564	4,173,436
Jupiter	4,320,000	364,224	3,955,776
Mars	4,320,000	2,296,824	2,023,176
Venus	4,320,000	4,320,000	2,702,388
Mercury	4,320,000	4,320,000	13,617,020

In fact, the numbers the Indians text quote for Venus and Mercury are the number of *heliocentric* revolutions for each planet in 4,320,000 years:

Venus: 7,022,388 = 4,320,000 + 2,702,388
Mercury 17,937,020 = 4,320,000 + 13,617,020

eccentric (*manda*) $\sin q(\alpha) = -e \sin \alpha$

epicycles (*sighra*) $\tan p(\gamma) = \dfrac{r \sin \gamma}{1 + r \cos \gamma}$

(1) $\alpha = \bar{\lambda} - \lambda_A \qquad \nu_1 = \bar{\lambda} + \tfrac{1}{2} q(\alpha)$

(2) $\gamma = \bar{\lambda}_S - \nu_1 \qquad \nu_2 = \nu_1 + \tfrac{1}{2} p(\gamma)$

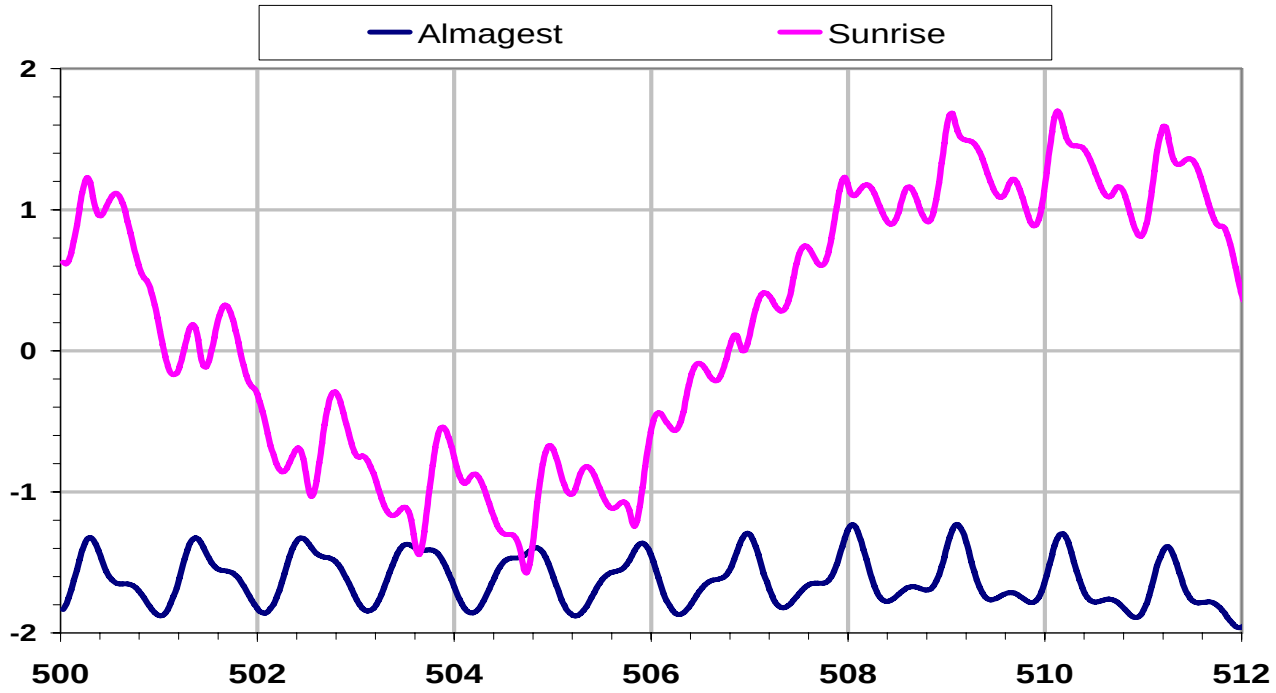
(3) $\alpha = \nu_2 - \lambda_A \qquad \nu_3 = \bar{\lambda} + q(\alpha)$

(4) $\gamma = \bar{\lambda}_S - \nu_3 \qquad \lambda = \nu_3 + p(\gamma)$

Aryabhata's text says:

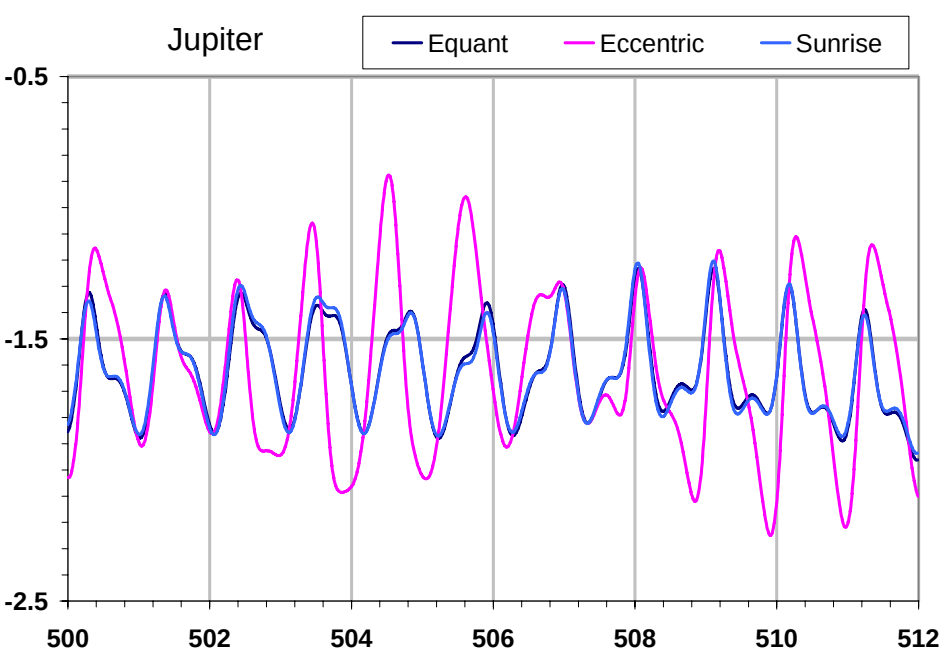
half the *mandaphala* obtained from the apsis is minus and plus to the mean planet. Half from the *sigraphala* is minus and plus to the manda planets. From the apsis they become *sphutamadhya* [true-mean]. From the *sigraphala* they become *sphuta* [true].

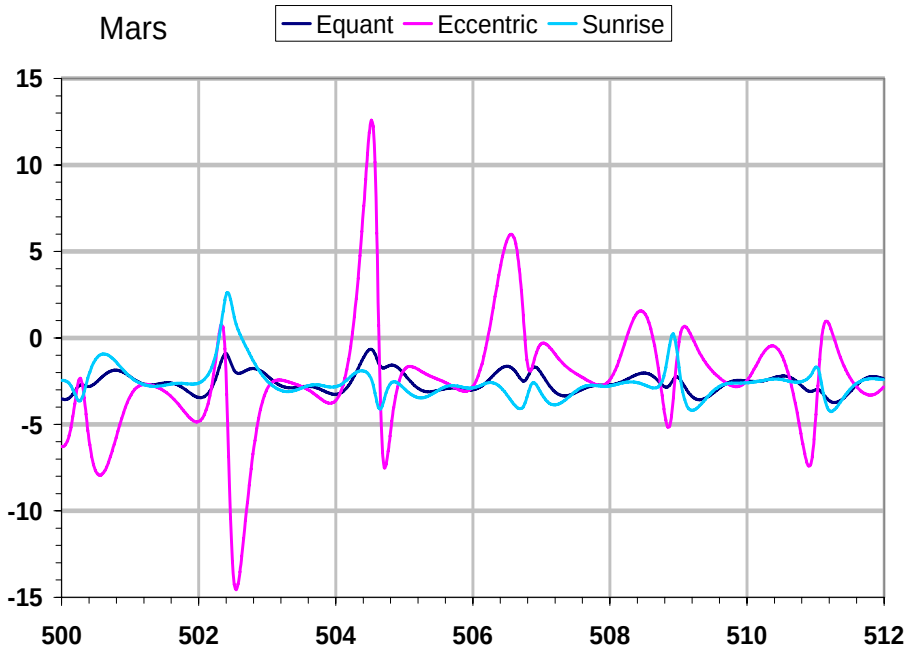
Jupiter



Most of the difference is due to poor orbit parameters in the Sunrise model.

What happens if we use *identical* orbit elements in both models?





Therefore, it is clear that the *Almagest* equant and the Indian models **share the same mathematical basis**.

Arabic astronomers were very unhappy with the equant since it violates Aristotle's principle of uniform motion in a circle. By about A.D. 1250 they had developed several alternatives that are as good as the equant and use only uniform motion.

http://people.scs.fsu.edu/~dduke/arabmars.html

The same issues bothered Copernicus (*ca*. 1520-1540) and he used the same models, although we do not know how he learned about them.