Ancient Astronomy

Lecture 3

February 14, 2007

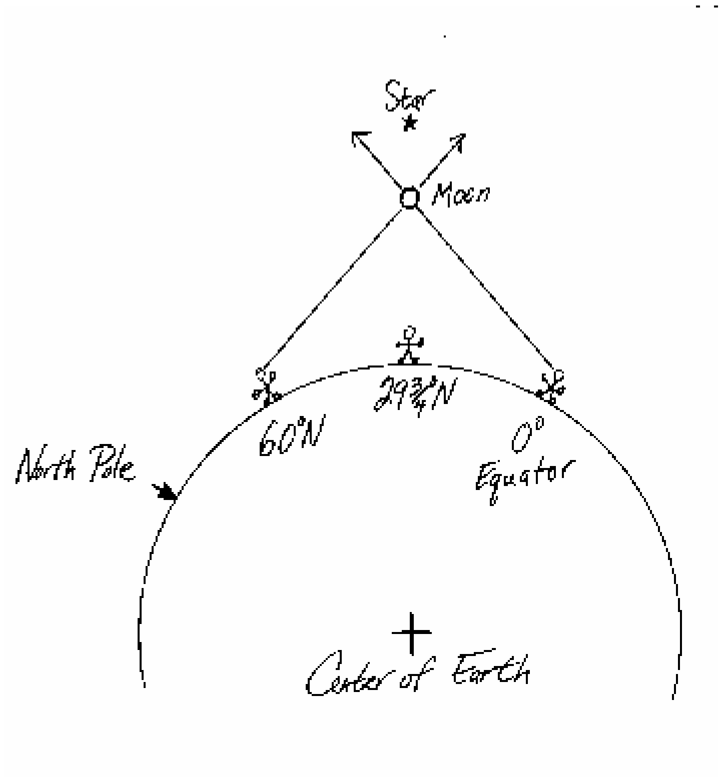
Course website**:** www.scs.fsu.edu/~dduke/lectures

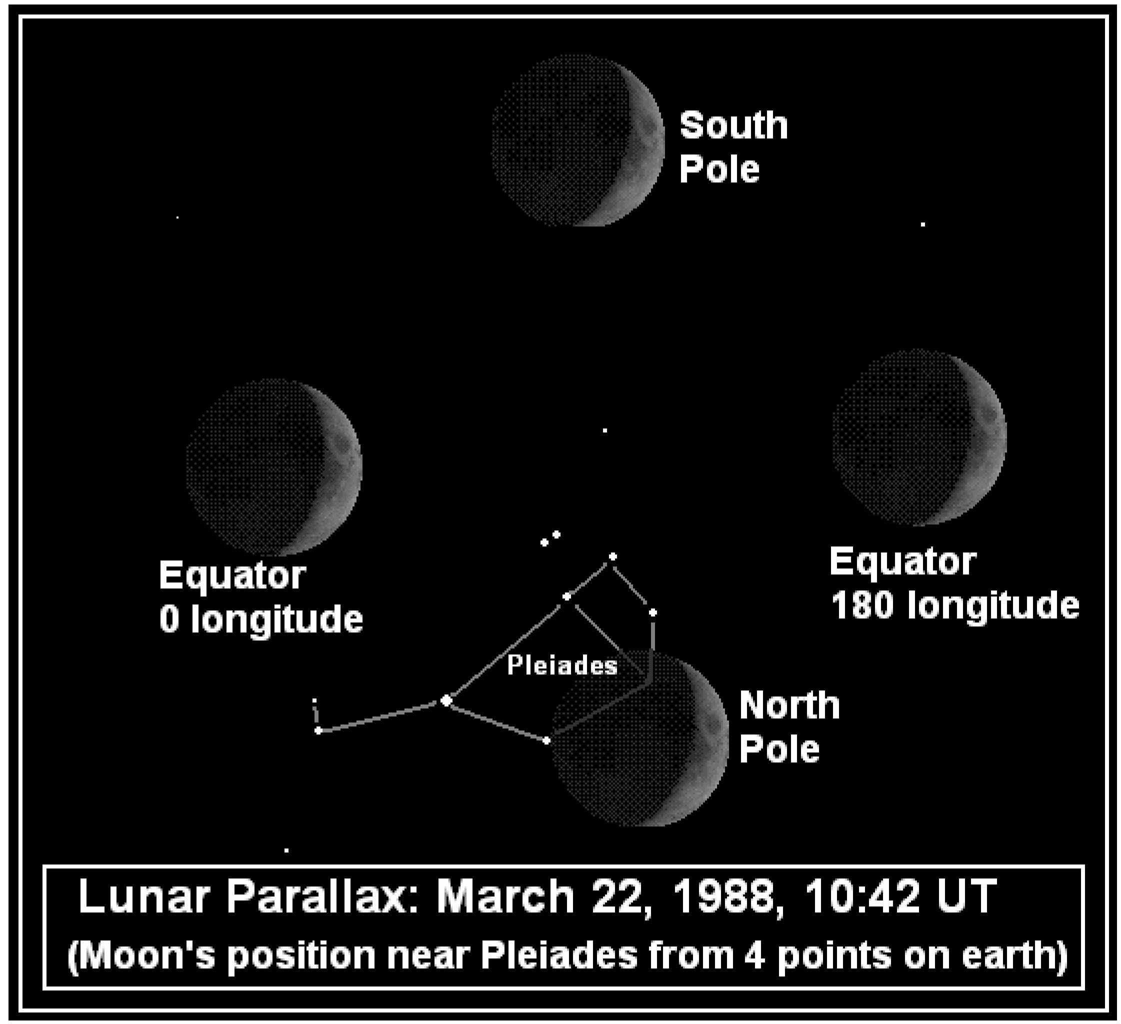
Lecture 3

- *Almagest* Books 4 – 6
- the Moon
- the problem of parallax
- the length of the various months
- the first geometric model
- the second geometric model
- sizes and distances of the Sun and Moon
- the background

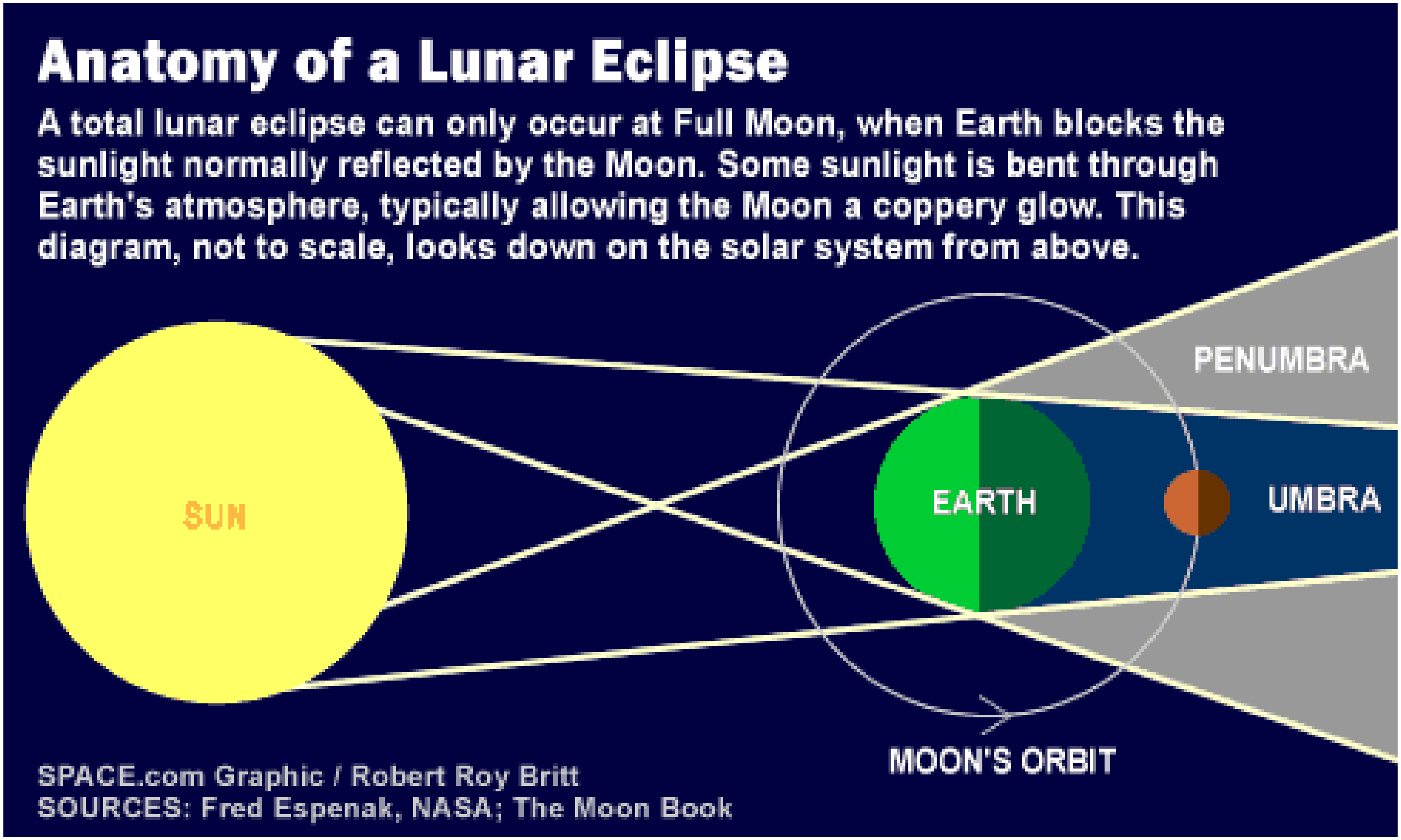
the Moon and the Sun are both about the same size as viewed from Earth: they both subtend about ½° in the sky.

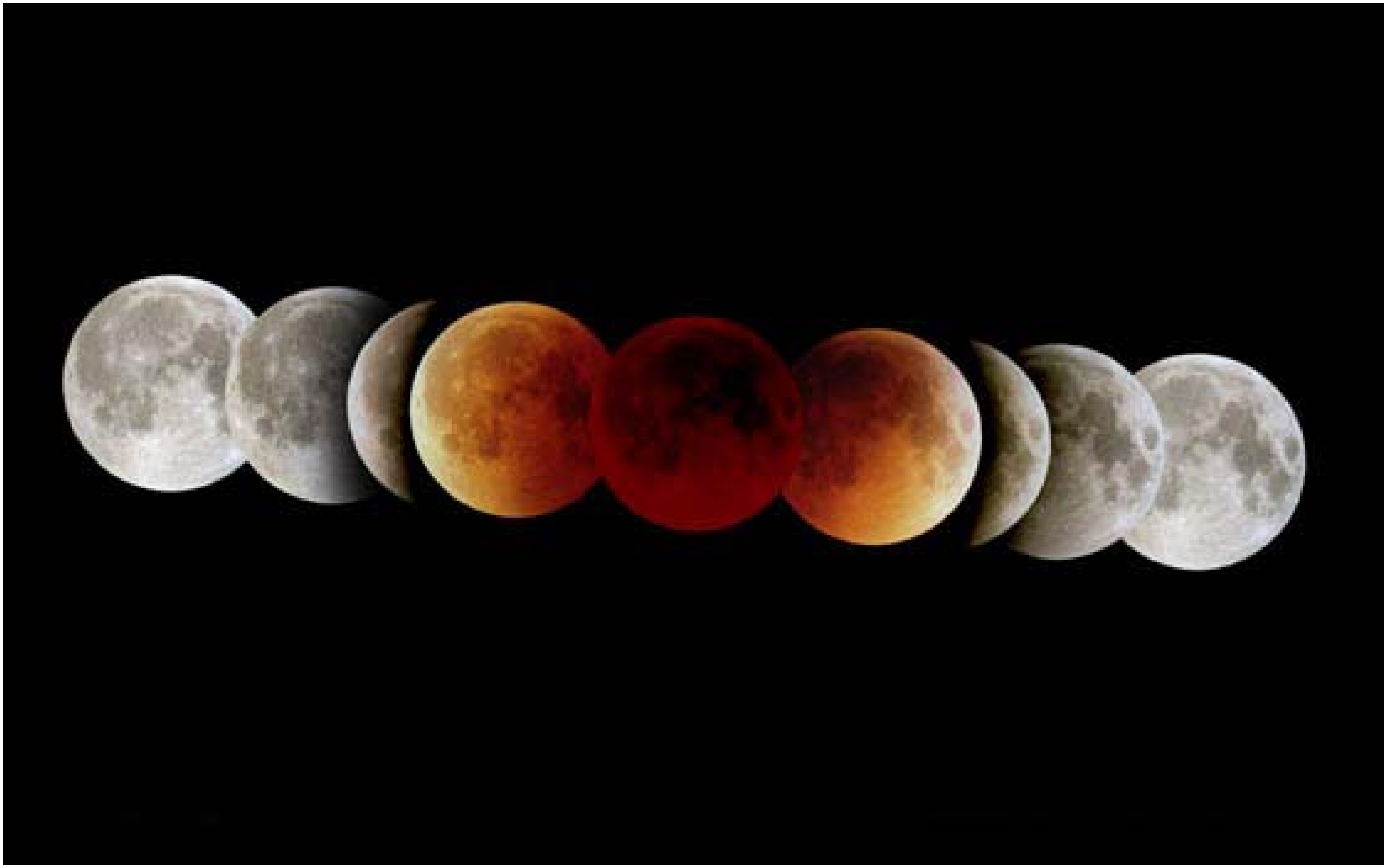
the distance to the Moon is *not* negligible compared to the size of the Earth.





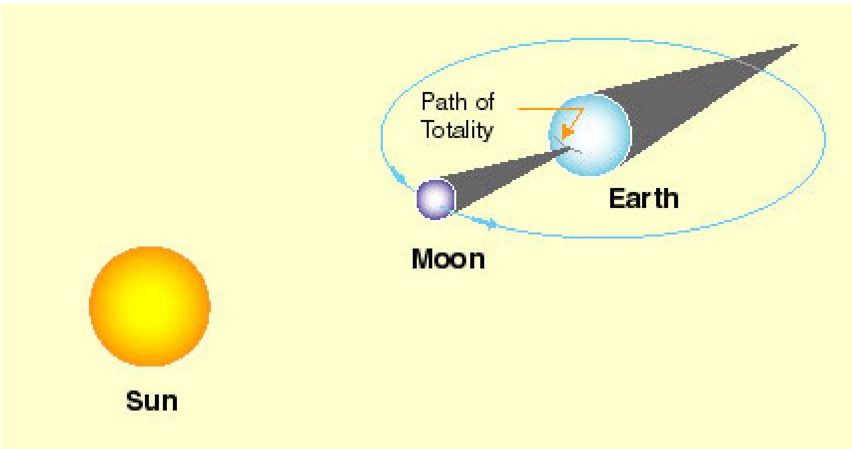
best observations are *times* of lunar eclipses: at that time we can *compute* the position of the Sun, and we then know that the Moon is exactly 180° away.

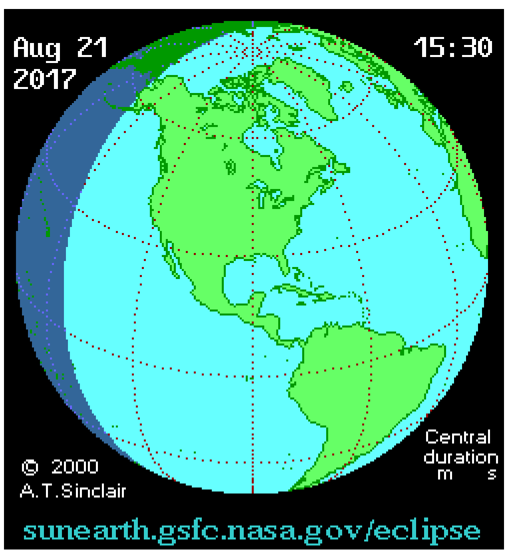


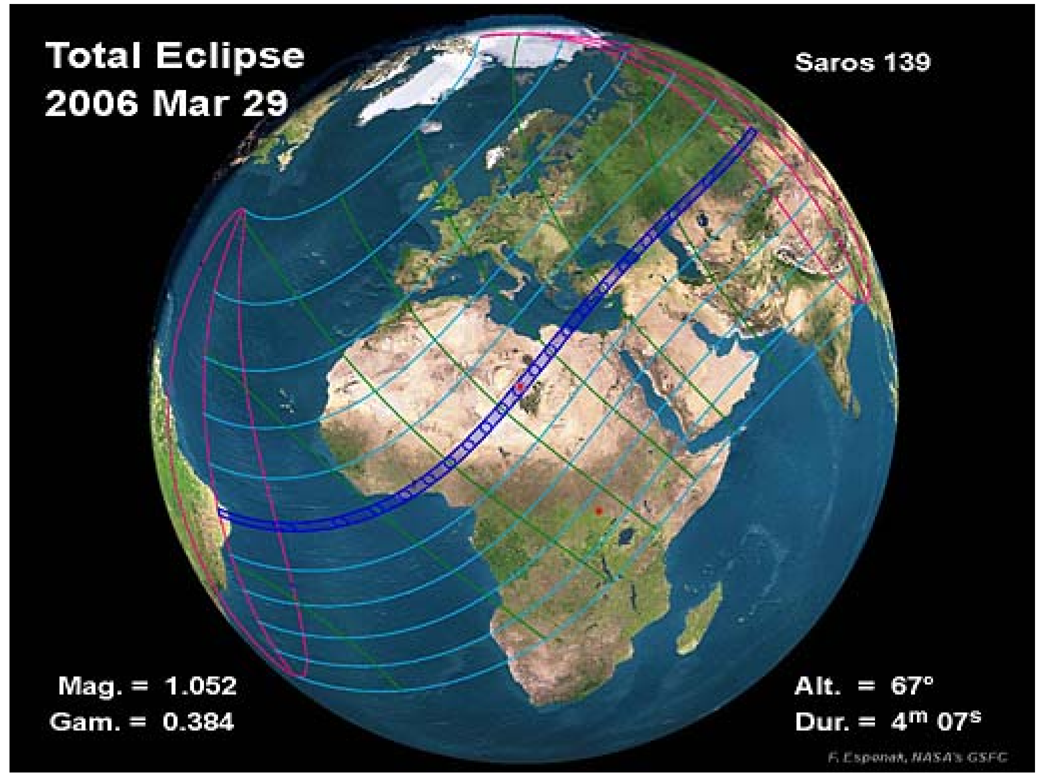


by the way, a solar eclipse is similar but a bit more complicated.

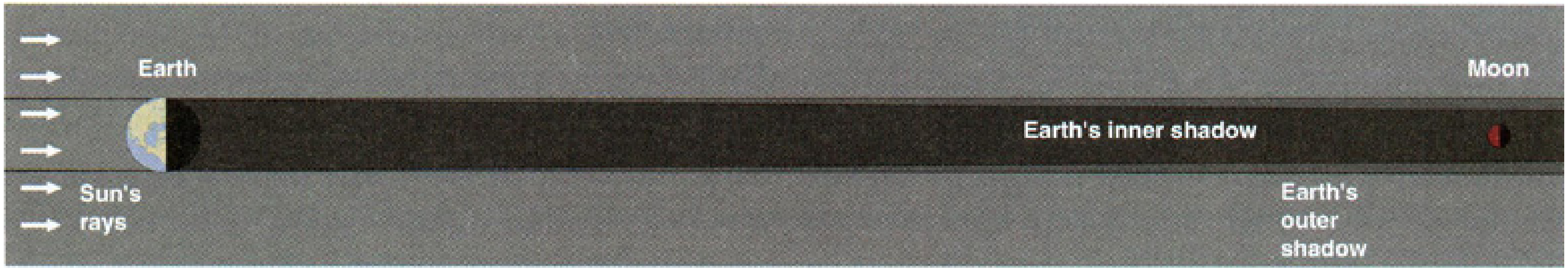
http://sunearth.gsfc.nasa.gov/eclipse/SEanimate/SE2001/SE2017Aug21T.GIF

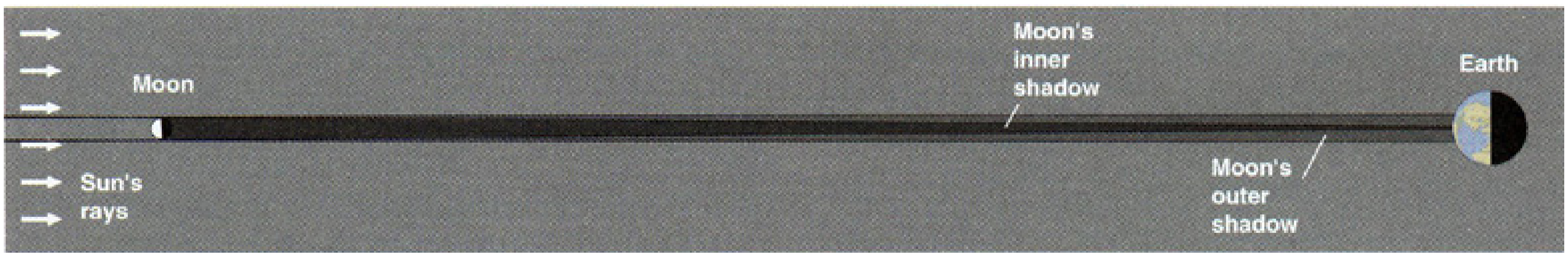






drawn to scale:





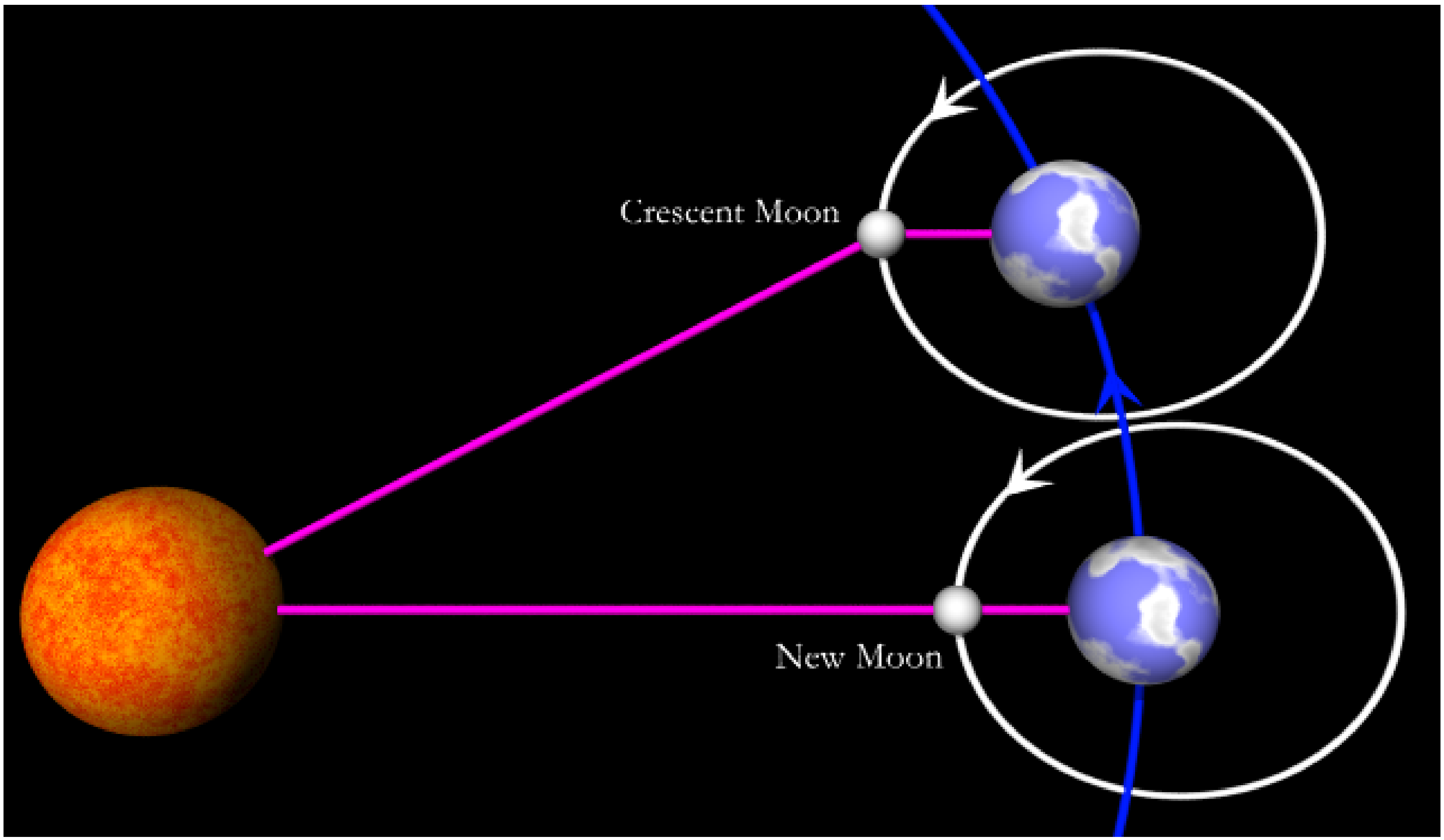
What is a ‘month’?

There are several:

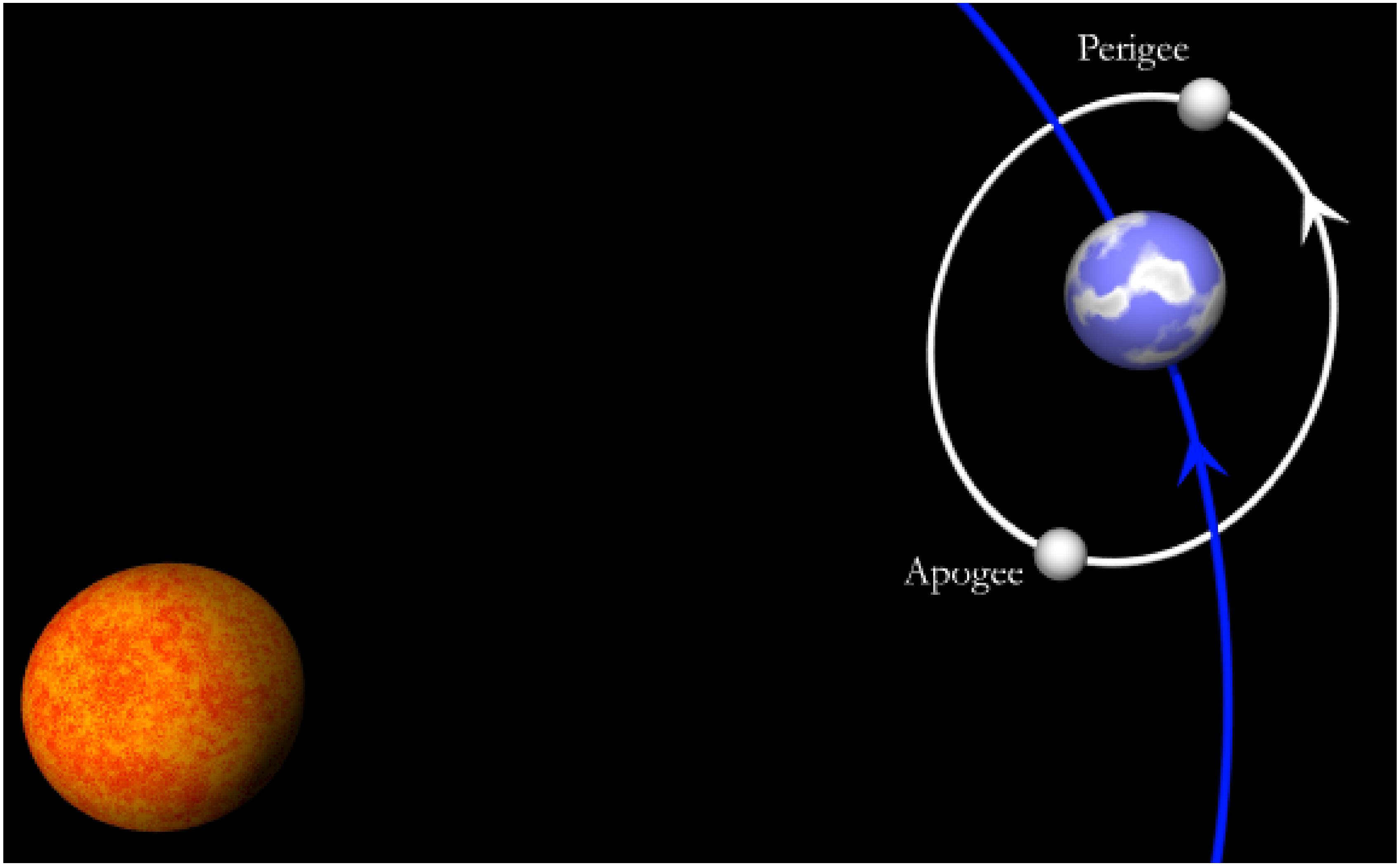
(a)	return to the same star on the ecliptic (sidereal).	$27^{d}\,07^{h}\,43^{m}\,12^{s}$
(b)	return to the same declination δ (tropical).	$27^{d}\,07^{h}\,43^{m}\,05^{s}$
(c)	return to the same speed (anomalistic).	$27^{d}\,13^{h}\,18^{m}\,33^{s}$
(d)	return to the same latitude (draconitic).	$27^{d}\,05^{h}\,05^{m}\,36^{s}$
(e)	return to the same angle from the Sun (synodic).	$29^{d}\,12^{h}\,44^{m}\,03^{s}$

The synodic month – from one new moon or full moon to the next – is the one we use in daily conversation.

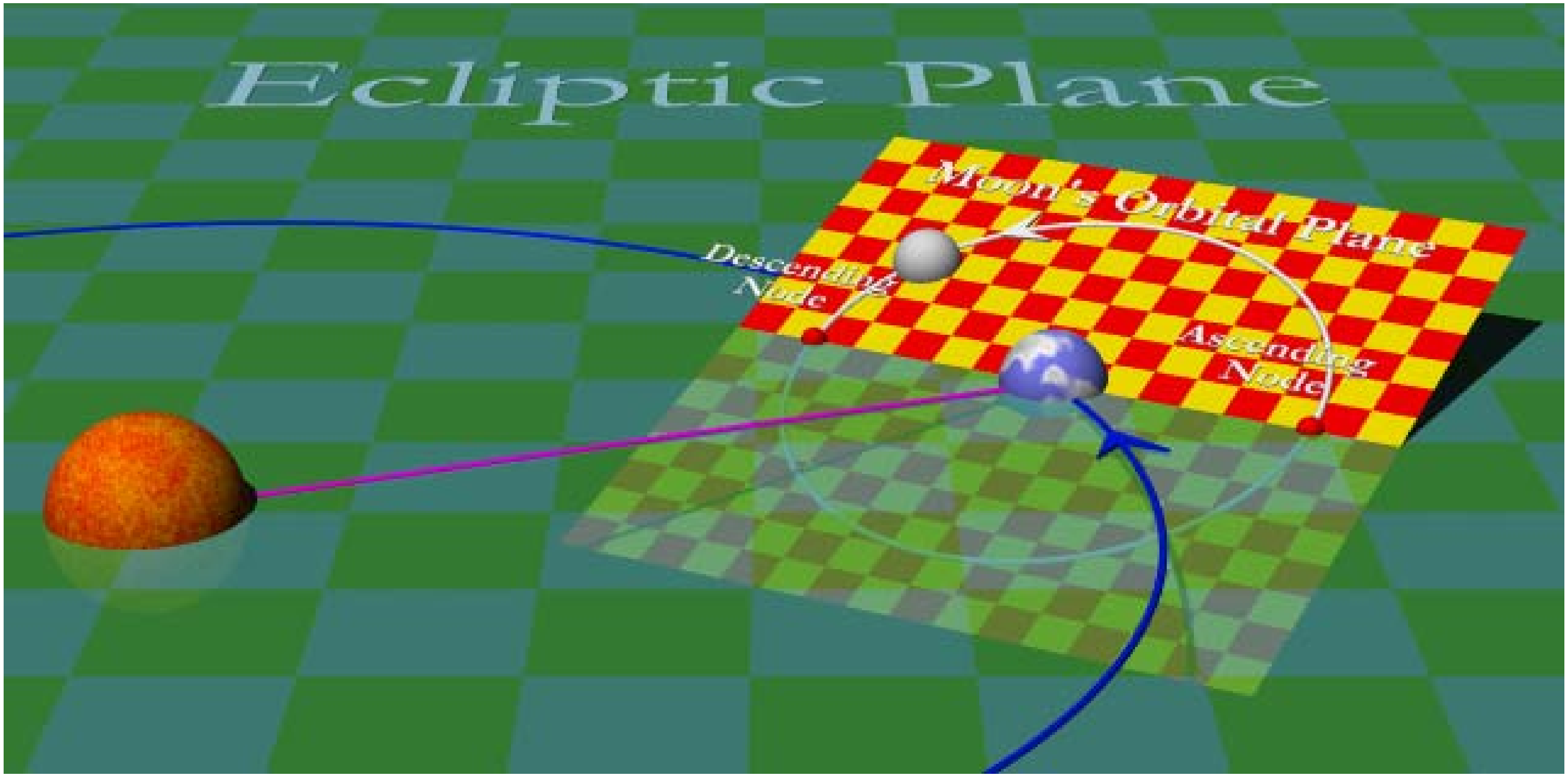
Sidereal Month (return to same longitude or fixed star)
Tropical Month (return to the same equinox or solstice)



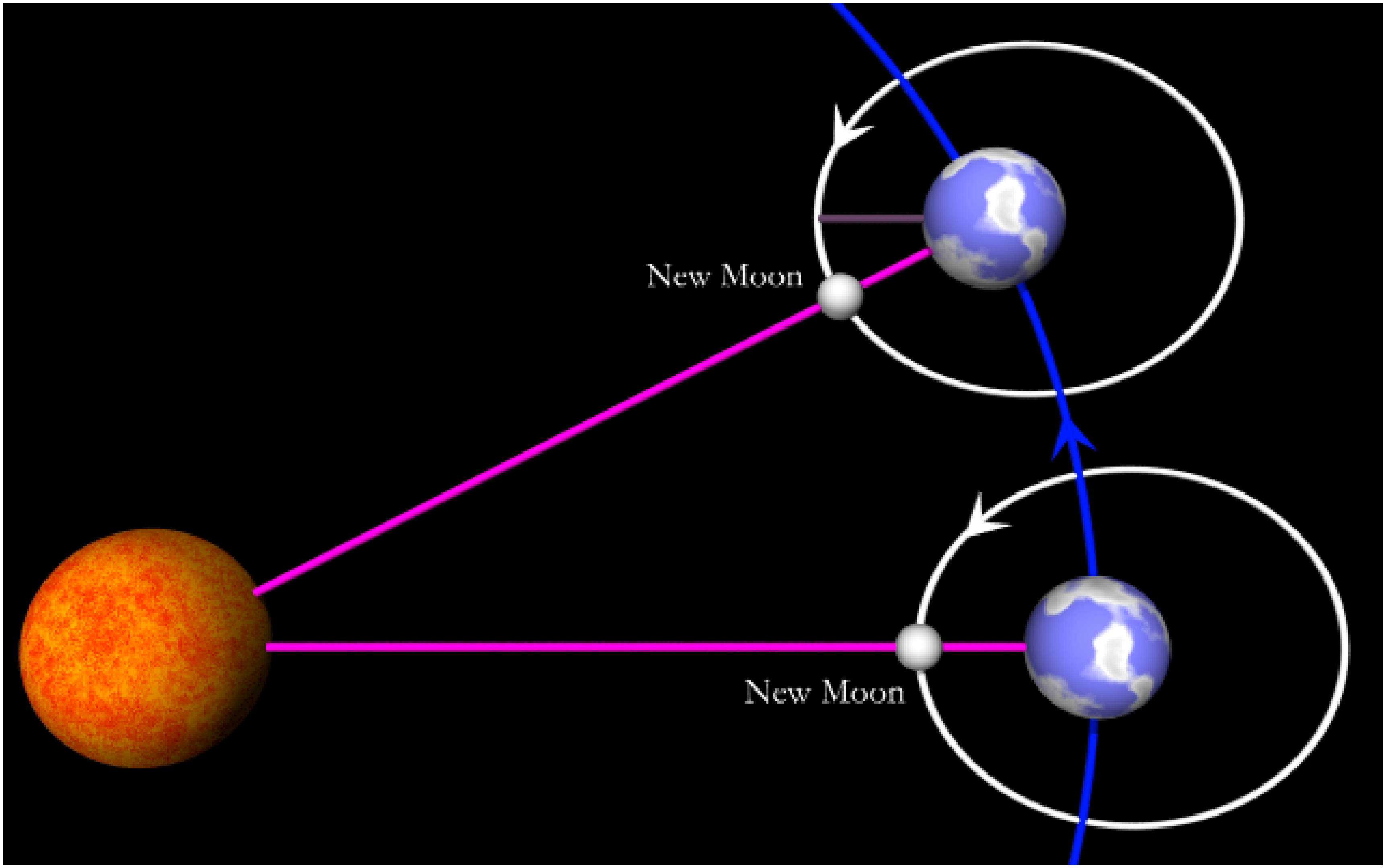
Anomalistic Month (return to same speed, e.g. fastest or slowest)

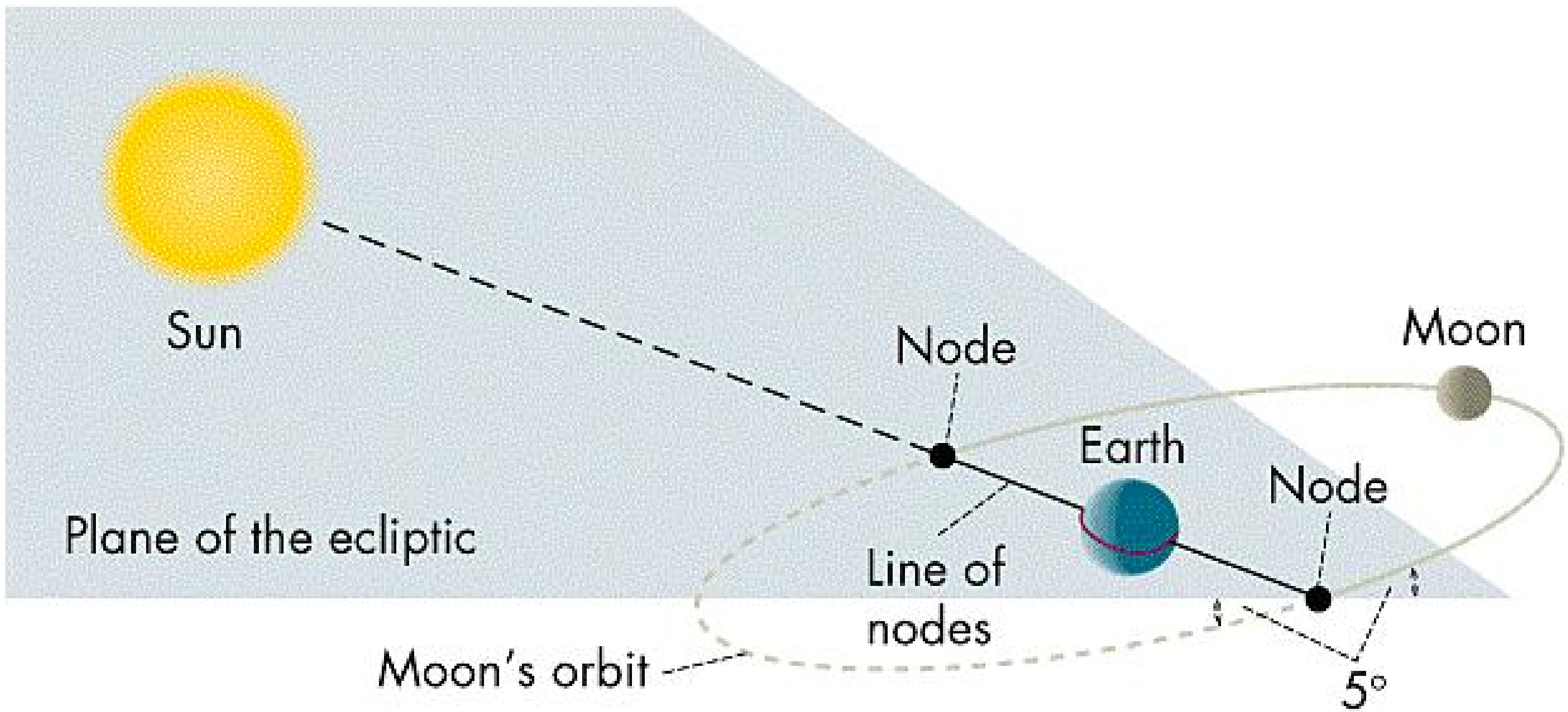


Draconitic Month (return to the nodes)

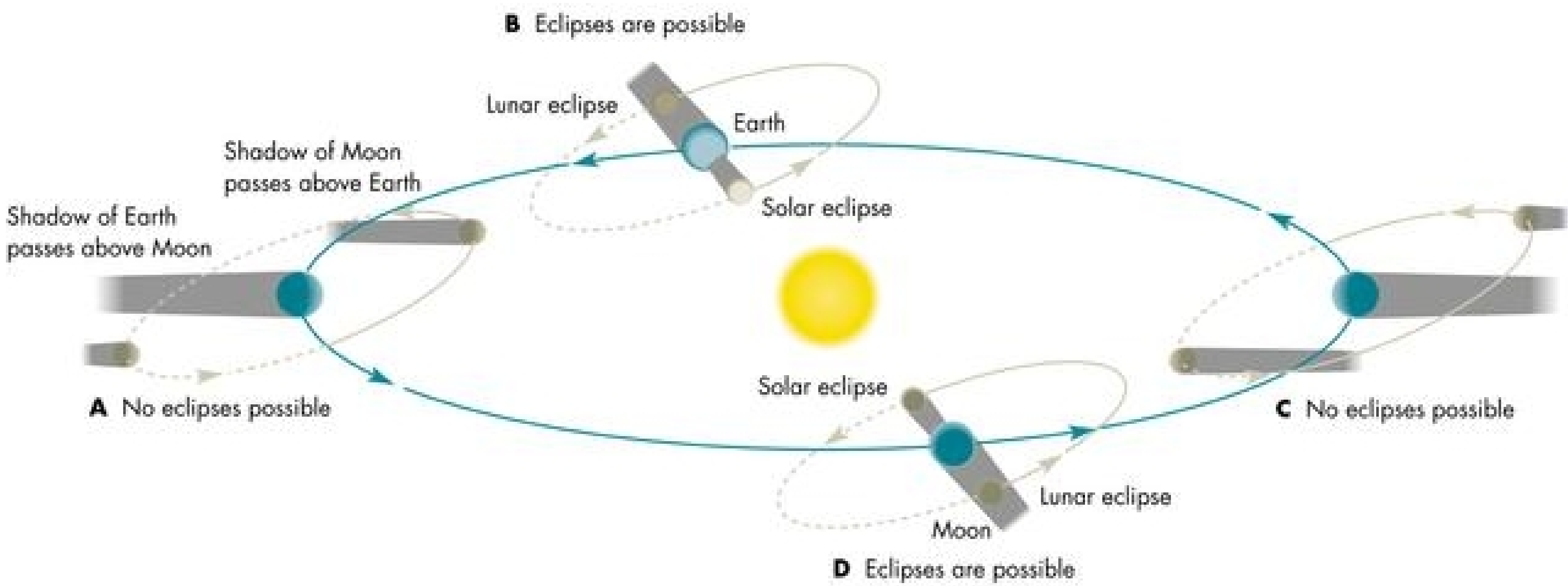


Synodic Month (return to the Sun)




Sun
Node
Moon
Earth
Node
Plane of the ecliptic
Line of
nodes
Moon's orbit
5°

to get an eclipse we must have the Sun-Earth-lunar nodes lines up, and the Moon fairly near a node (with about ±7°). On average we get about two eclipses per year, *somewhere*.



Period Relations

Periodic (Saros)
$6585\frac{1}{3}^d = 223^m = 239^a = 242^d = 241^t + 10\frac{2}{3}°$ (about 18^y)

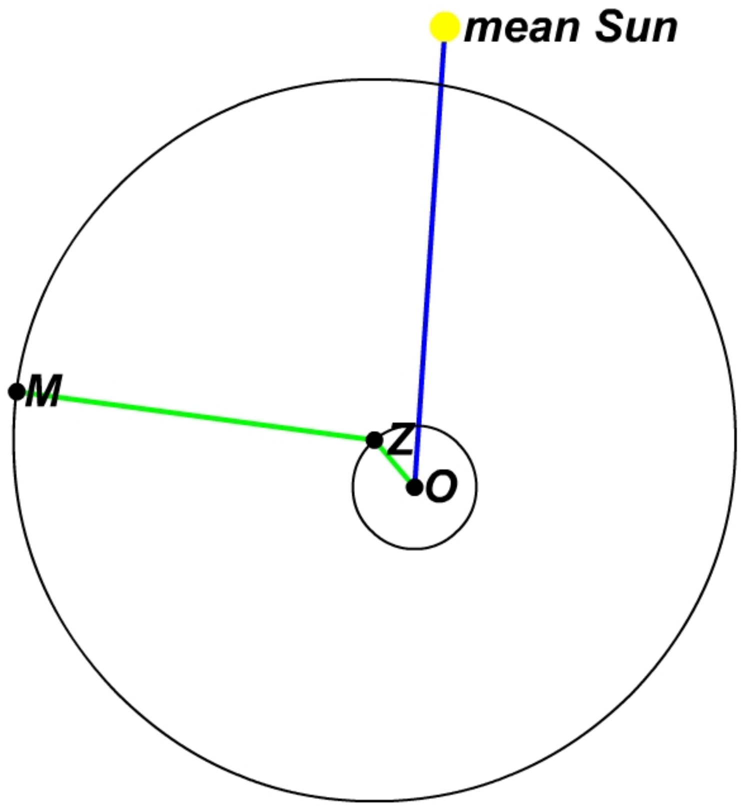
Exeligmos (3x Saros)
$19{,}756^d = 669^m = 717^a = 726^d = 723^t + 32°$ (about 54^y)

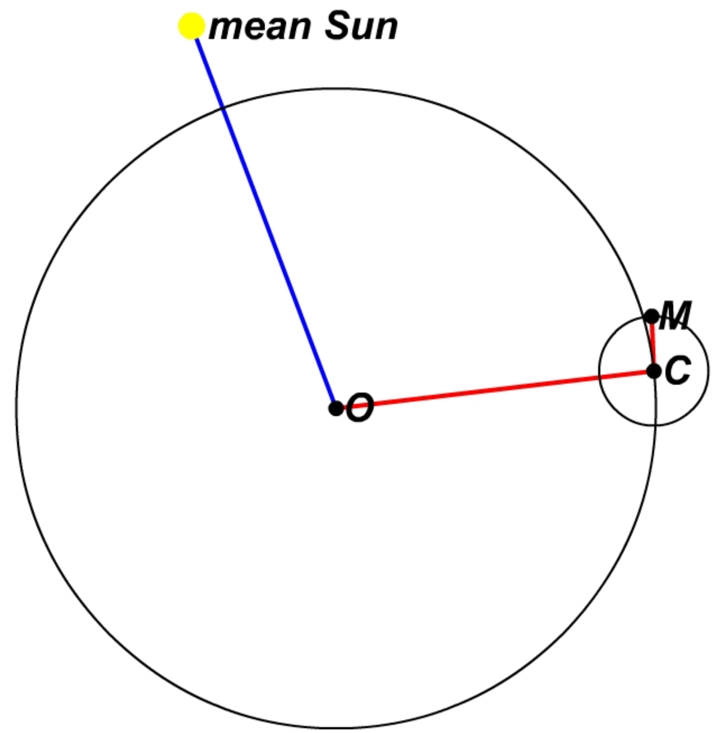
Hipparchus (Babylonian)
$126{,}007^d\ 1^h = 4267^m = 4573^a = 726^d = 4612^t - 7\frac{1}{2}°$ (about 345^y)
and $5458^m = 5923^d$

Note that $126007^d\ 1^h / 4267^m = 29^d\ 12^h\ 44^m\ 02^s$ (compared to $29^d\ 12^h\ 44^m\ 03^s$)

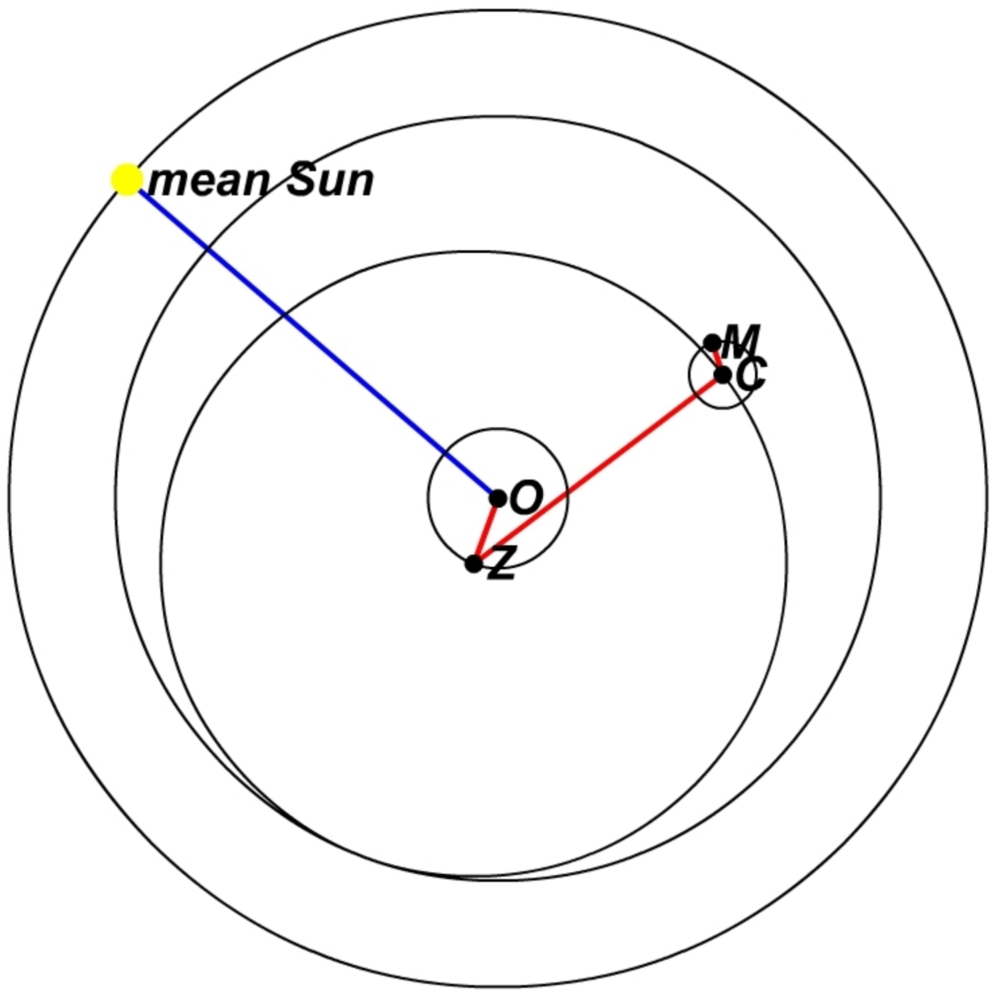
All of these come from centuries of eclipse records in Babylon, starting around 750 B.C. if not earlier (remember that Alexander the Great conquered Babylon in 323 B.C.)

Ptolemy and Hipparchus found that regarding just new moon and full moon, when the Sun and Moon are in a line with the Earth, a simple model would work.
http://www.scs.fsu.edu/~dduke/models.htm



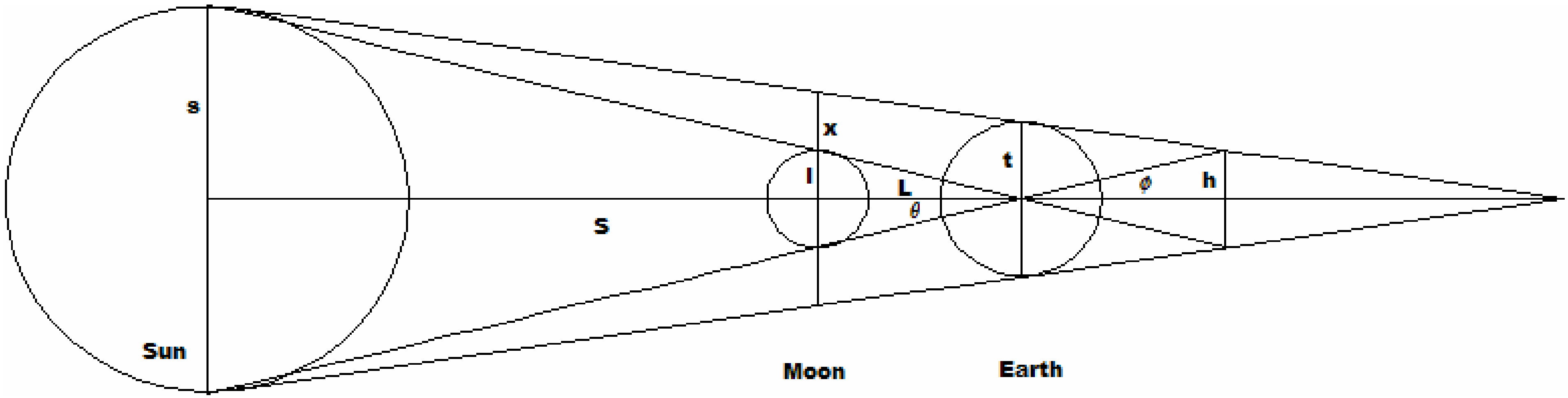


However, in the more general case the simple model fails and Ptolemy uses a more complicated model. http://www.scs.fsu.edu/~dduke/models.htm



Sizes and Distances of the Sun and Moon

Ptolemy gives an analysis which is extremely delicate to compute.



$$S \approx \frac{L}{\left(\frac{\varphi}{\theta}+1\right) L \sin\theta - 1} = 1/\left(\left(\frac{\varphi}{\theta}+1\right)\sin\theta - \frac{1}{L}\right)$$

Ptolemy takes
$\theta = 0;31,20°$
$\varphi = 2\ 3/5\ \theta = 1;21,20°$
$L = 64;10$

to get $S = 1210$

but for example $\varphi/\theta = 2\ 2/5$ makes $S < 0$

The Background

Ptolemy’s usual fudging

Luni-Solar calendars

Babylonian models

Ptolemy's fudging

for the simple model he produces two trios of lunar eclipses:

-720 Mar 19/20	7:30 pm		133 May	6/7	11:15 pm
-719 Mar 8/9	11:10 pm		134 Oct	20/21	11:00 pm
-719 Sep 1/2	8:30 pm		136 Mar	5/6	4:00 am

Analysis of both trios gives virtually identical results, and changing any of the times by even a few minutes substantially changes the results.

Later, in *Almagest* 4.11, he gives two more trios and once again gets the very same answers. Such coincidences are very unlikely.

for the complicated model

(a) Ptolemy wants to know the *maximum* angle the true Moon can differ from the average Moon. In the case of the simple model this is 5°. Ptolemy produces two observations which he analyzes to get a maximum angle of 7;40° (in both cases). But he neglected parallax, and if he had included it he would have gotten 7;31° and 7;49° for the two cases.

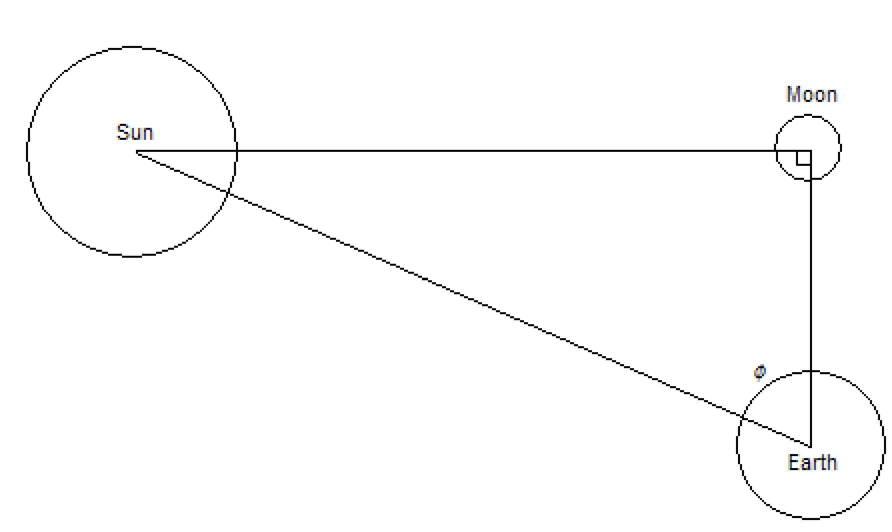
(b) Ptolemy needs to know the size of his new central epicycle, so he produces two observation that both give him 10;19. In both cases he miscomputes but still manages to get the same answer.

(c) Ptolemy's complicated model makes the apparent size of the Moon vary by almost a factor two. In reality it varies by about 15% (maximum to minimum).

for the sizes and distances Ptolemy has to very carefully analyze eclipses from 523 B.C. and 621 B.C. (why so ancient?). In the end he finds

$$\frac{S}{L} \simeq 19$$

However, in about 240 B.C. Aristarchus, in a completely different kind of analysis, also found $S/L \simeq 19$. He assumed only that the angle Moon-Earth-Sun was 87° at half-moon.



In between Ptolemy and Aristarchus, Hipparchus used slightly different parameters to get a much different answer:

Ptolemy takes
$\theta = 0;15,40°$
$\varphi = 2\ 3/5\ \theta = 0;40,40°$
$L = 64;10$

to get $S = 1210$

Hipparchus takes
$\theta = 0;16,37°$
$\varphi = 2\ 1/2\ \theta = 0;41,33°$
$L = 67;20$

to get $S = 490$, or he assumed $S = 490$ and computed $L = 67;20$

the correct answers are about $L = 60$ and $S = 23{,}000$

Luni-Solar Calendars

The fact that the month is just a bit longer than 29½ days caused a lot of bother in establishing a workable calendar that keeps months properly aligned with the year and its seasons.

Early try: Meton and Euctomen (about 430 B.C.): the Metonic calendar

19 years = 235 months = 6940 days
= 12 years of 12 months plus 7 years of 13 months

There are 125 full (30-day) months and 110 hollow (29-day) months

Resulting year is 365 5/19 days

Resulting month is 29 + 1/2 + 3/94 days

365 5/9 is longer than 365 ¼ by 1/76 day. Hence Callippus (about 330 B.C.) suggested a new calendar with four successive 19-year Metonic cycles but leaving out 1 day from one of the cycles:

76 years = 940 months = 27,759 days
= 4 x 235 months = 4 x 6,940 days – 1 day

Resulting year is 365 1/4 days

Resulting month is 29 + 1/2 + 29/940 days

The fraction 29/940 is about 1/32.4 whereas a slightly more accurate value is 1/33, and this was known to Geminus and hence would have been widely known.

There may have been even older and simpler calendars. Geminus describes ones with 8 years = 99 months and 16 years = 198 months and 160 years = 1979 months. In all of these either the month or year length is not good enough.

The Antikythera Mechanism

In 1900, a team of Greek sponge-divers working off the islet of Antikythera, midway between the Peloponnese and Crete, discovered an antique shipwreck 42 metres below the surface of the Mediterranean Sea. Among the many objects they recovered from the wreck, which has been dated to around 65 BC, were several bronze fragments. At first overlooked, these were later associated with some sort of astronomical machinery. But the realization that this was the earliest-known device involving an arrangement of gear-wheels came only slowly. In fact, staggeringly, the Antikythera Mechanism is the most sophisticated such object yet found from the ancient and medieval periods.

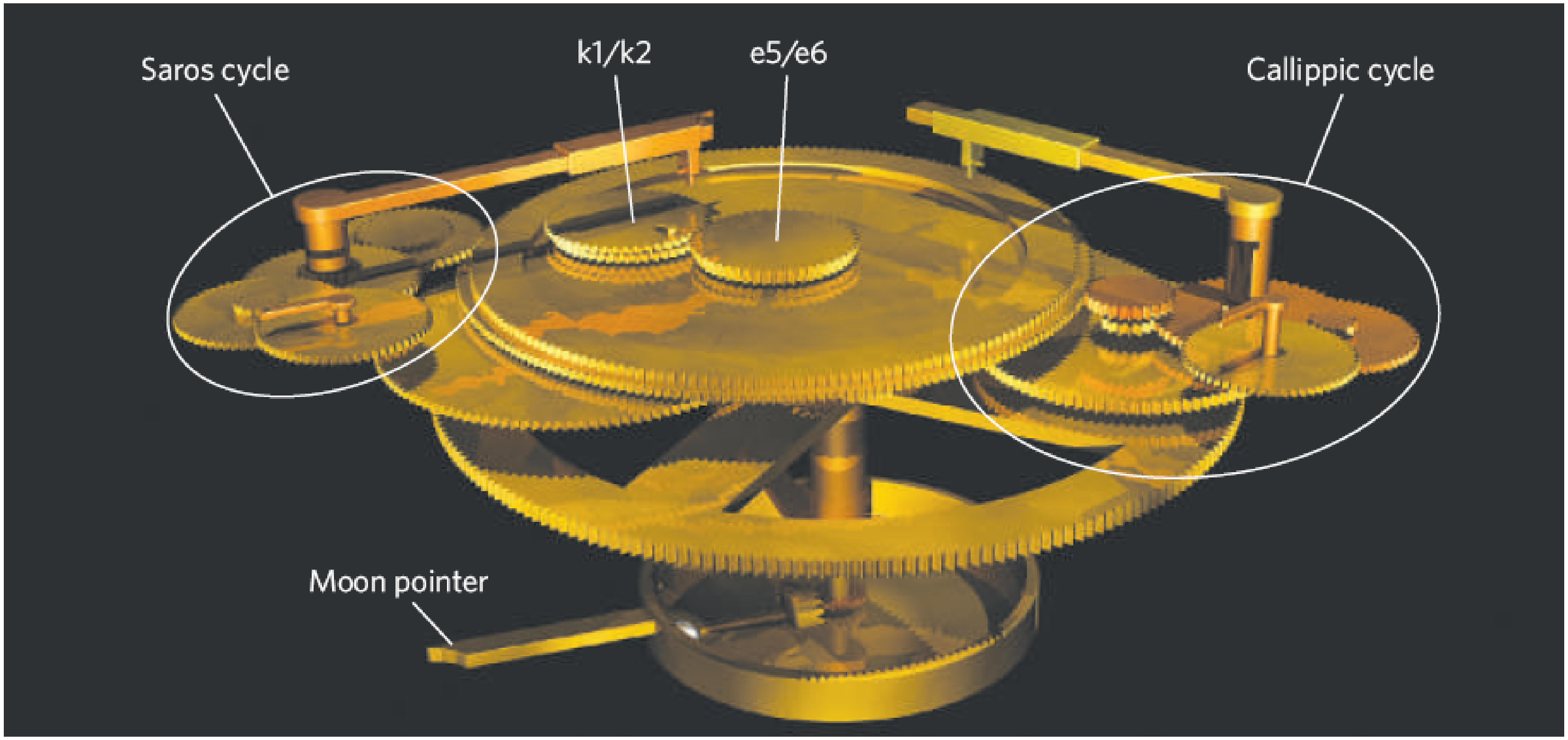
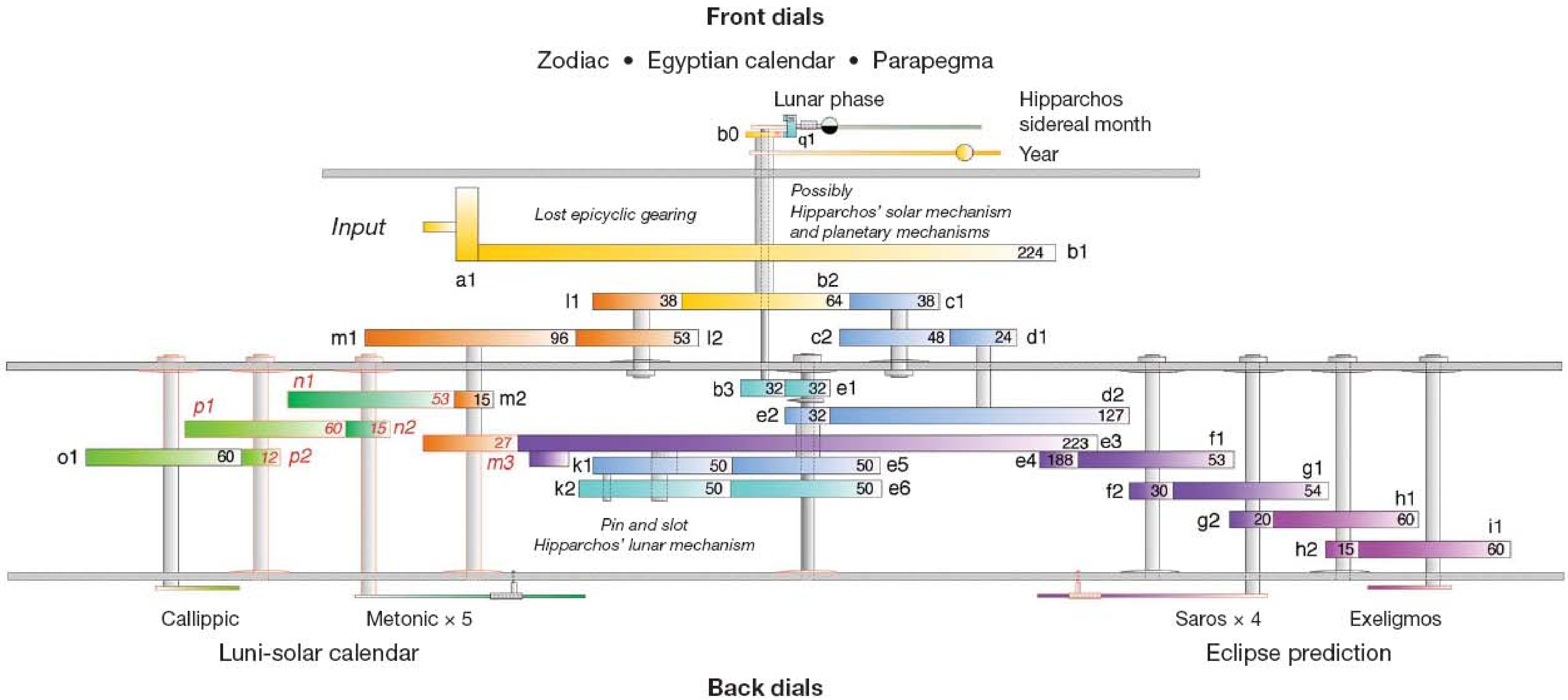
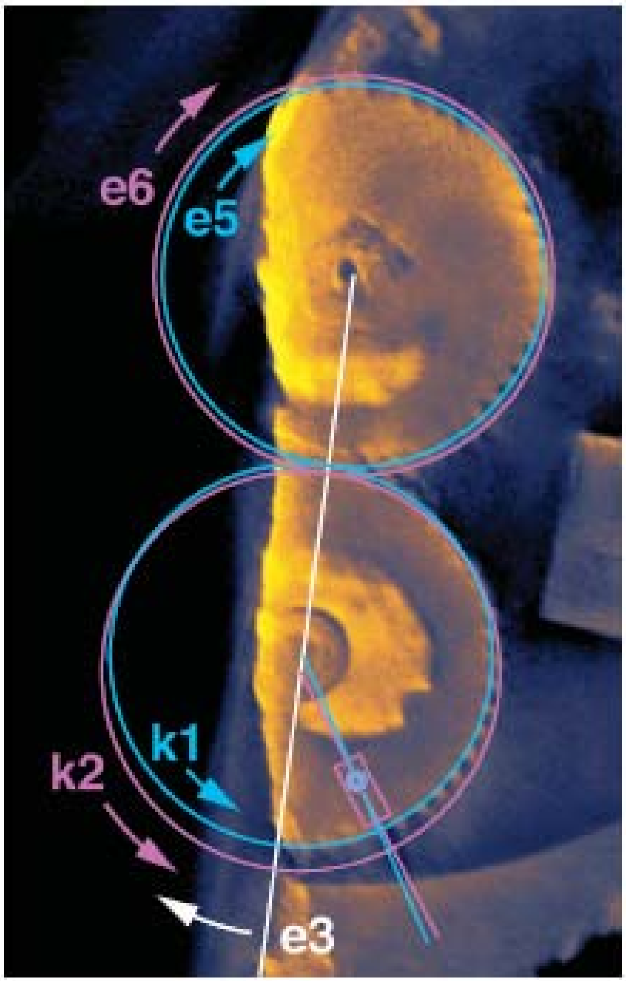


Figure 1 | Wheels within wheels. The rear side of Freeth and colleagues' reconstruction[1] of the Antikythera Mechanism, viewed sideways on. The left gear and pointer system simulated the Saros cycle for predicting lunar and solar eclipses; the right gears and pointers were for the Callippic cycle that synchronizes synodic months and solar years. At the centre, mounted on the large gear-wheel, were two pairs of identical gear-wheels, e5/e6 at the centre and k1/k2 at the left (see also Fig. 5 on page 590). The pair k1/k2 was provided with a pin-and-slot device that induced an irregular movement in the pointer at the front of the mechanism indicating the position of the Moon. This system simulated a model of the Moon's motion developed by Hipparchus of Rhodes in the second century BC.

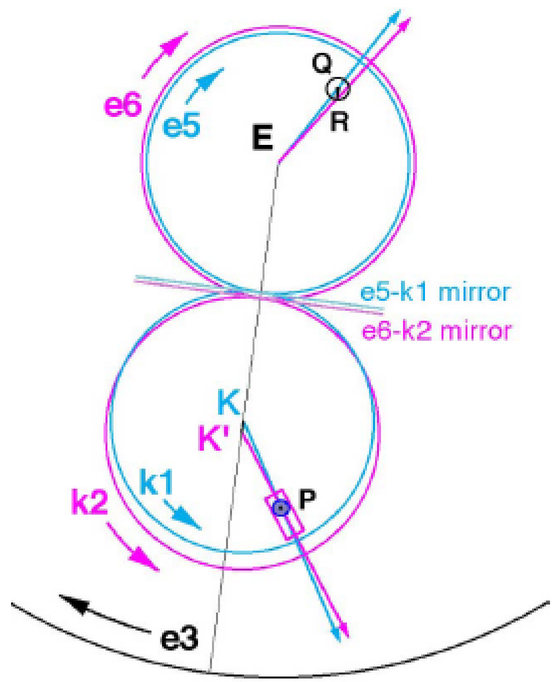

Front dials
Zodiac • Egyptian calendar • Parapegma
Lunar phase
Hipparchos sidereal month
Year
b0
q1
Possibly Hipparchos' solar mechanism and planetary mechanisms
Lost epicyclic gearing
Input
224
b1
a1
b2
l1
38
64
38
c1
m1
96
53
l2
c2
48
24
d1
n1
53
15
m2
b3
32
32
e1
d2
p1
60
15
n2
e2
32
127
27
223
e3
f1
o1
60
12
p2
m3
k1
50
50
e5
e4
188
53
g1
k2
50
50
e6
f2
30
54
h1
Pin and slot Hipparchos' lunar mechanism
g2
20
60
i1
h2
15
60
Callippic
Metonic × 5
Saros × 4
Exeligmos
Luni-solar calendar
Eclipse prediction
Back dials

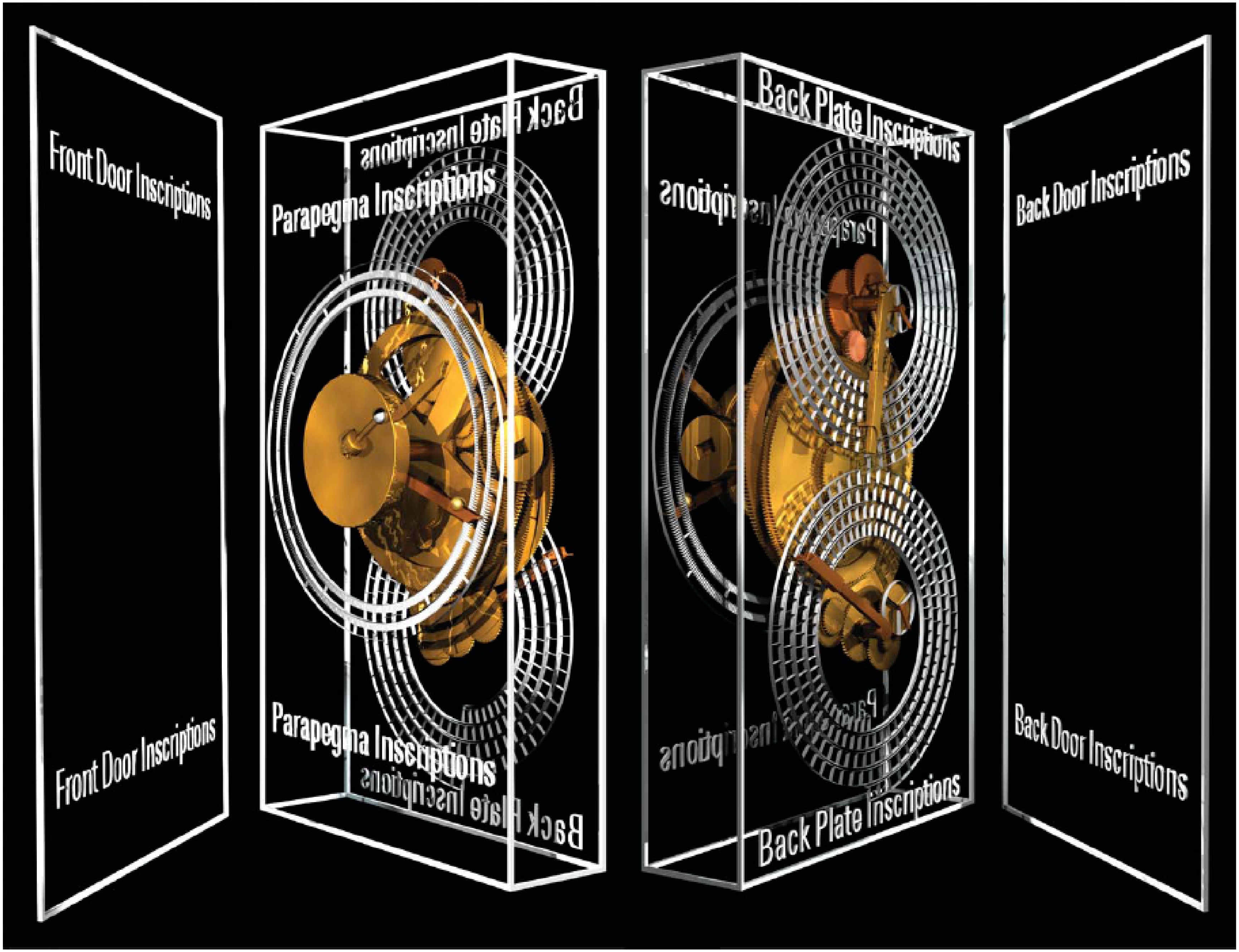
the pin-and-slot mechanism to simulate Hipparchus' model



Geometric Proof

We now also give a geometric proof using elementary methods, and so in principle accessible in ancient Greece, establishing that the pin-and-slot mechanism is equivalent to Hipparchos' epicyclic lunar theory.

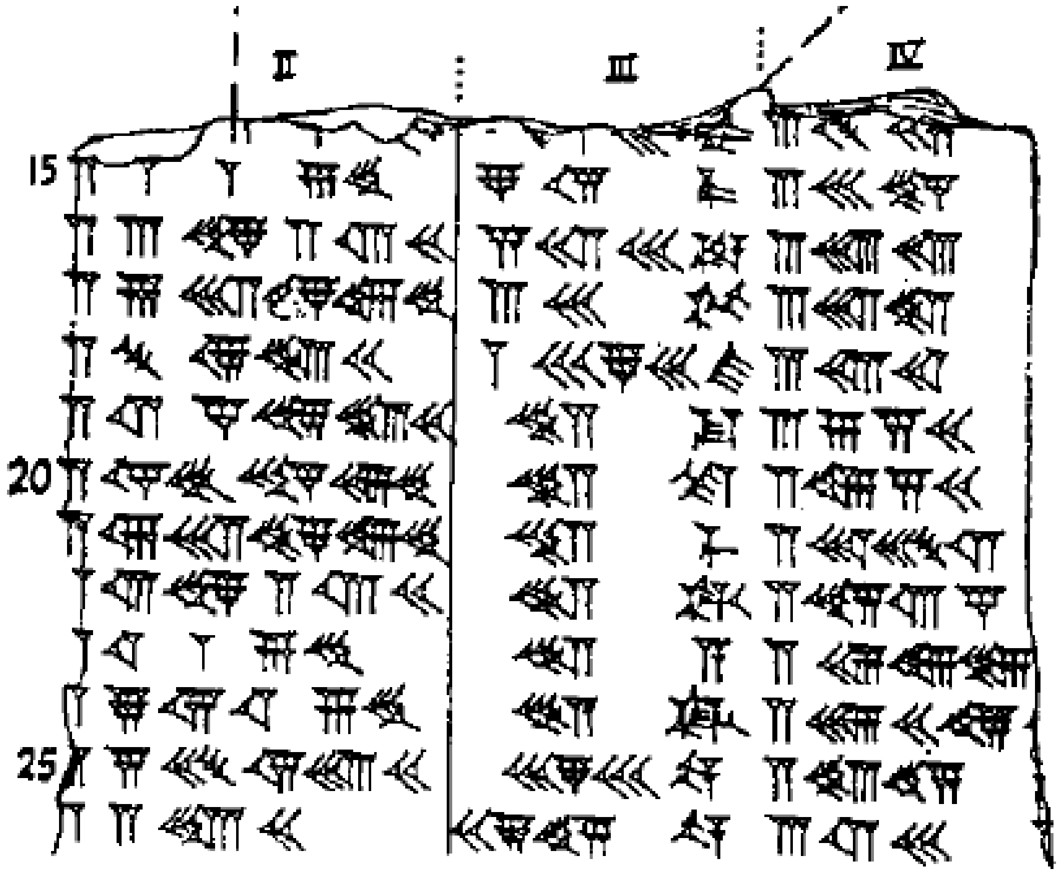



Front Door Inscriptions
Parapegma Inscriptions
Parapegma Inscriptions
Front Door Inscriptions
Back Plate Inscriptions
Back Plate Inscriptions
Back Plate Inscriptions
Back Plate Inscriptions
Back Door Inscriptions
Back Door Inscriptions

Babylonian Astronomy

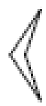
During the late 1800's some 50,000 or so clay tablets were sent to the British Museum from Babylon and Uruk.

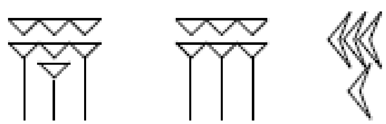
About 250 of the tablets related to astronomy were studied by two Jesuit priests, Fathers Epping and Strassmaier in the late 1800's and followed by Father Kugler in the early 1900's.



Babylonian numerals:

 represents 1

 represents 10



could mean 7 + 6/60 + 40/3600
(i.e. 7 1/9 or 7.1111...)
written 7;6,40

or it could mean 7 times 60 + 6 + 40/60
(i.e. 426 2/3 or 426.6666...)
written 7,6;40

The work of the three Fathers revealed a previously unsuspected history of very involved mathematical astronomy developed in Babylon starting about 450 B.C.

Before their work science in Babylon was generally associated with ideas like magic, mysticism, and astrology. These people were often referred to as the Chaldeans.

Whereas the Greek models were designed to give the position of the Sun or Moon at any moment in time, the Babylonians were interested in predicting the times and position of sequences of quasi-periodic events – new moon, full moon, etc.

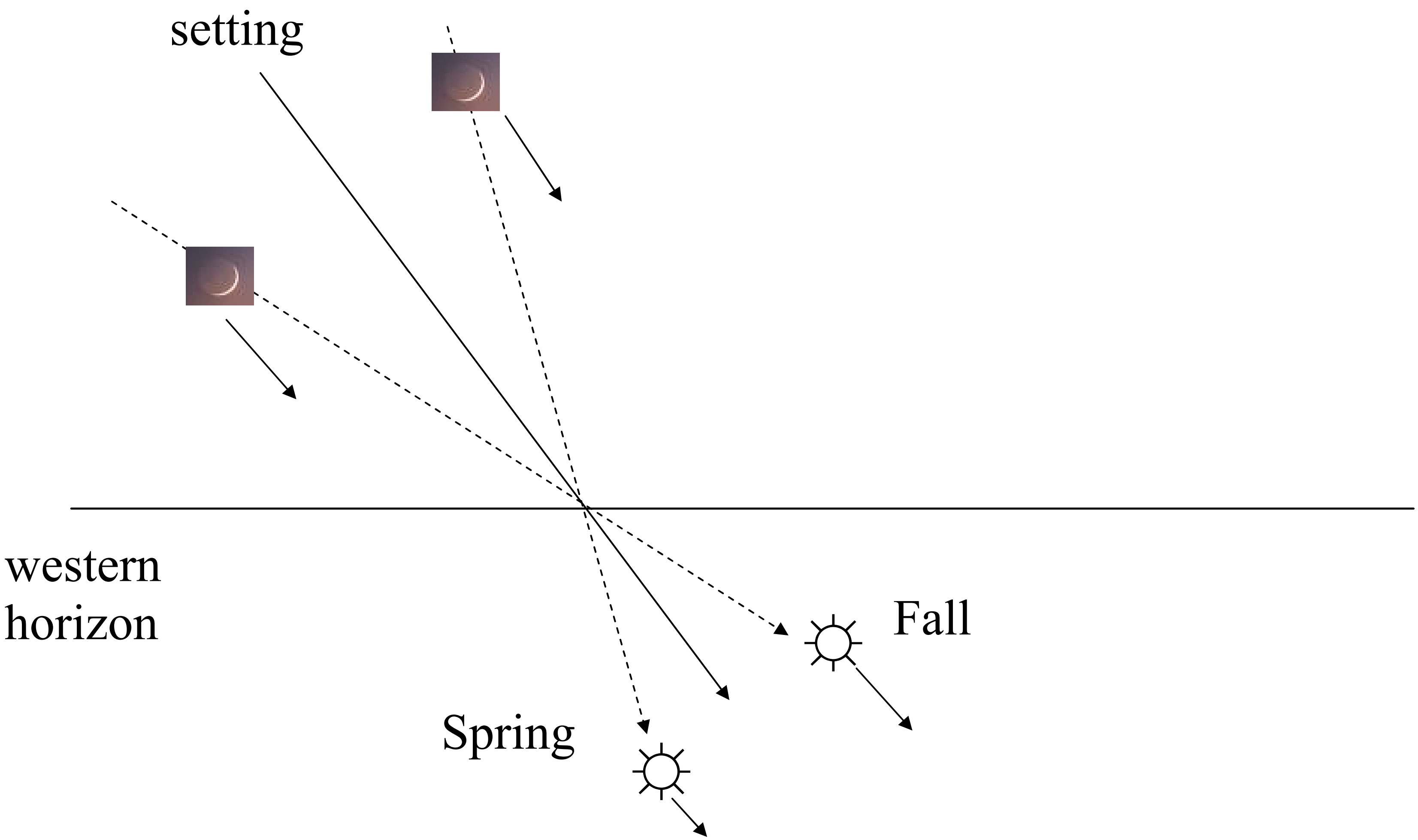
The Babylonians used a purely lunar calendar. The “lunar month” begins on the evening when the lunar crescent is ***first visible*** shortly after sunset.

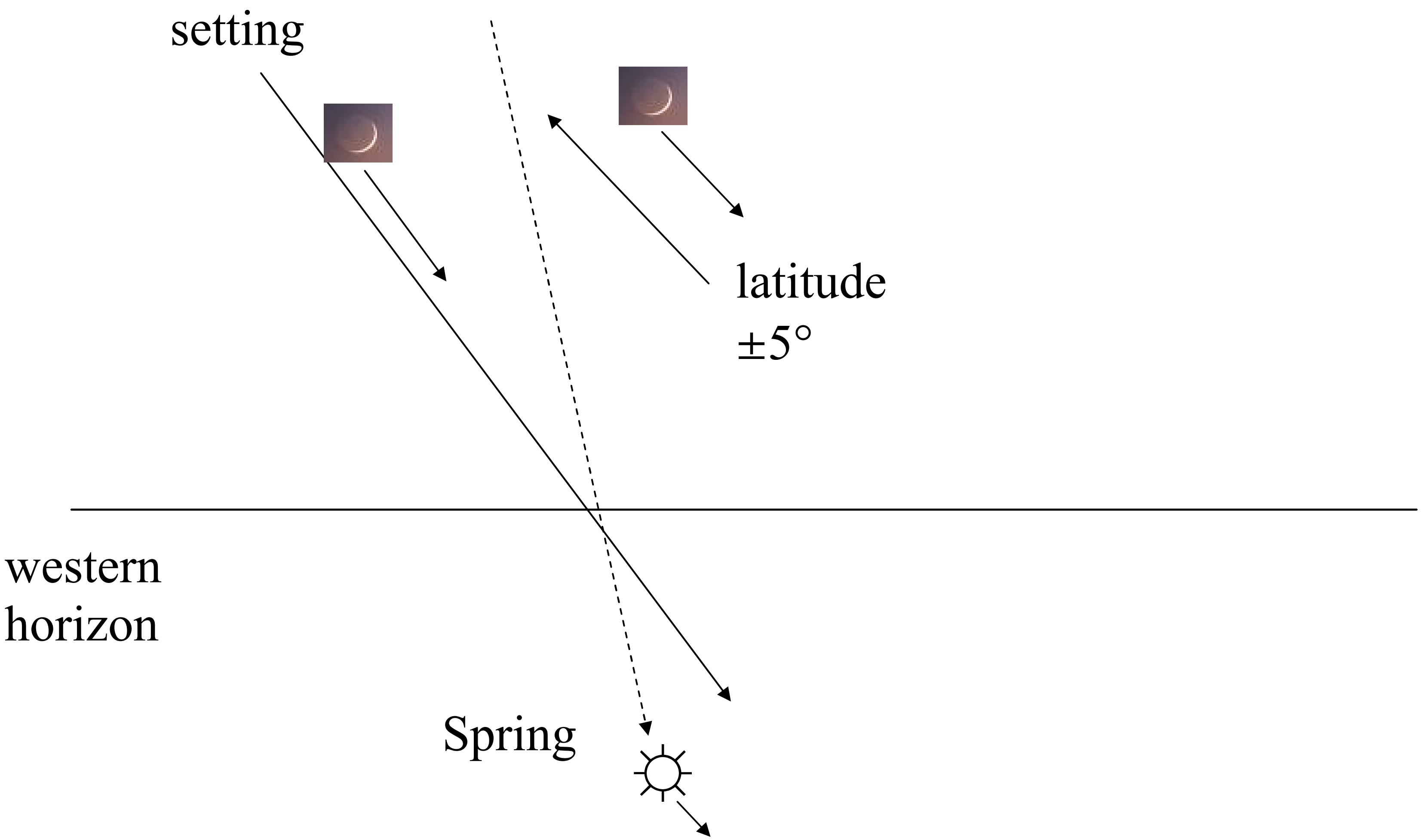
Such a definition has a number of intrinsic difficulties, and Babylonian lunar theory was developed to deal with these complications.

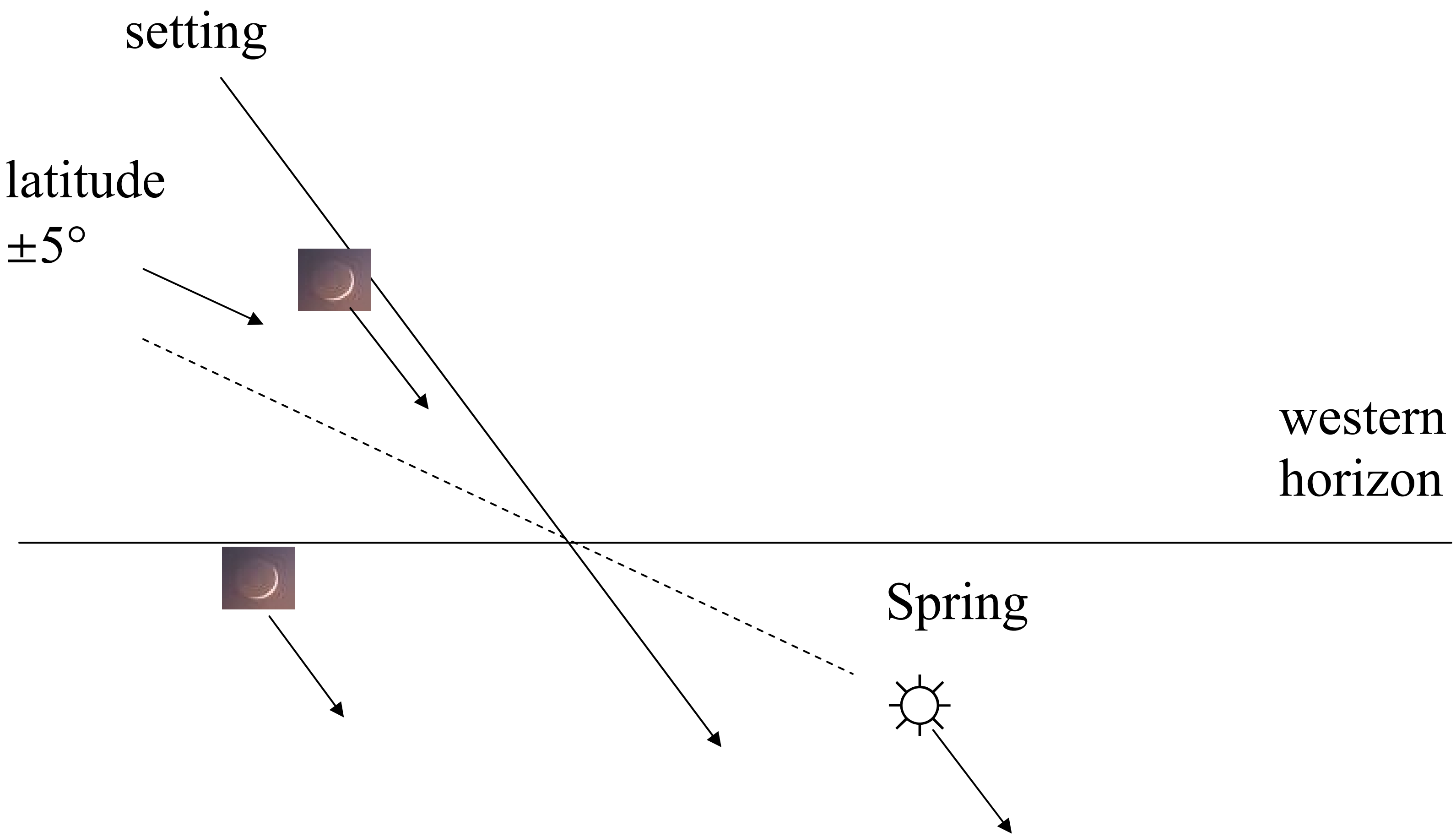
How many days are in a “lunar month”? Each such month is either 29 or 30 days, but we need to know which *in advance*.

This clearly involves both the varying speed of the Moon and the varying speed of the Sun. Remember that the Moon covers about 13° per day and the Sun about 1° per day, but these are *averages*. So we must account for the departure from average throughout each month.

There are seasonal changes due to the angle between the ecliptic and the horizon and also changes due to the varying latitude of the Moon.


setting
western
horizon
Fall
Spring


setting
latitude
±5°
western
horizon
Spring


setting
latitude
±5°
western
horizon
Spring

The ‘astronomical diaries’ were kept for many centuries and are night-by-night accounts of where the various celestial objects were to be found:

No. -567

1 Year 37 of Nebukadnezar, king of Babylon. Month I, (the 1st of which was identical with) the 30th (of the preceding month), the moon became visible behind the Bull of Heaven; [sunset to moonset:] .... [....]

2 Saturn was in front of the Swallow. The 2nd, in the morning, a rainbow stretched in the west. Night of the 3rd, the moon was 2 cubits in front of [....]

3 it rained'. Night of the 9th (error for: 8th), beginning of the night, the moon stood 1 cubit in front of β Virginis. The 9th, the sun in the west [was surrounded] by a halo [.... The 11th]

4 or 12th, Jupiter's acronychal rising. On the 14th, one god was seen with the other; sunrise to moonset: 4°. The 15th, overcast. The 16th, Venus [....]

5 The 20th, in the morning, the sun was surrounded by a halo. Around noon, ..., rain PISAN. A rainbow stretched in the east. [....]

6 From the 8th of month XII_2 to the 28th, the river level rose 3 cubits and 8 fingers, 2/3 cubits [were missing] to the high flood [....]

7 were killed on order of the king. That month, a fox entered the city. Coughing and a little *rišūtu*-disease [....]

The result was a list of eclipses covering about six centuries, which Hipparchus apparently had access to.

In addition, the Babylonians kept extensive records of several centuries of observations of the times between rising/setting of the Moon and the Sun.

On the first day of the month:

(1) the time between sunset and the setting of the moon after it had become visible for the first time after conjunction. This interval is called NA.

Around the middle of the month, four intervals relating to full moon:

(2) the time between moonset and sunrise when the moon set for the last time before sunrise; this is called ŠÚ.

(3) the time between sunrise and moonset when the moon set for the first time after sunrise; called NA.

(4) the time between moonrise and sunset when the moon rose for the last time before sunset; called ME.

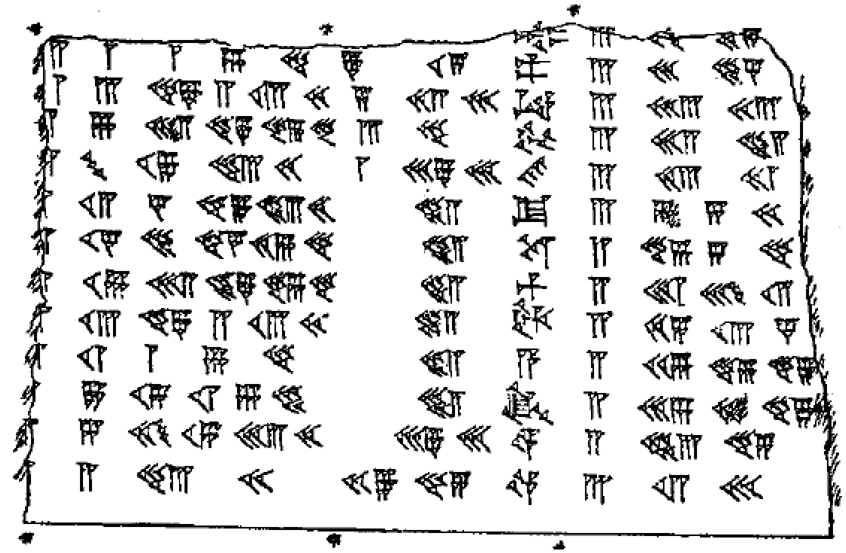
(5) the time between sunset and moonrise when the moon rose for the first time after sunset; called GE_6.

At the end of the month:

(6) the time between moonrise and sunrise when the moon was visible for the last time; called KUR.

The lunar theories

Each tablet is a set of columns of numbers



NEUGEBAUER's notation: *T Φ B C E Ψ F G J K L M*

(KUGLER's notation: *A B C D E F G H I K L M*)

ACT 13 (Reverse) = Sp II 110. Full Moons for the year 195

Col. T		Col. Φ	Col. B		Col. C
195	I	1, 58, 15, 11, 6, 40	9, 7, 30	(8)	3, 19, 25
	II	2, 1, 1, 6, 40	7, 15	(9)	3, 30, 54
	III	2, 3, 47, 2, 13, 20	5, 22, 30	(10)	3, 33, 23
	IV	2, 6, 32, 57, 46, 40	3, 30	(11)	3, 32, 52
	V	2, 9, 18, 53, 20	1, 37, 30	(12)	3, 23, 21
	VI	2, 12, 4, 48, 53, 20	52	(1)	3, 6, 5, 20
	VII	2, 14, 50, 44, 26, 40	52	(2)	2, 46, 5, 20
	VIII	2, 16, 32, 57, 46, 40	52	(3)	2, 31, 39, 12
	IX	2, 13, 47, 2, 13, 20	52	(4)	2, 25, 13, 4
	X	2, 11, 1, 6, 40	52	(5)	2, 26, 46, 56
	XI	2, 8, 15, 11, 6, 40	52	(6)	2, 36, 20, 48
	XII	2, 5, 29, 15, 33, 20	37, 30	(7)	2, 53, 45
196	I	2, 2, 43, 20	28, 45	(7)	3, 12, 30

Col. B: Longitude of Moon in signs and degrees.

Col. C: Duration of Daylight, in large hours.

Col. E: Latitude of Moon, in 'barleycorn'. The unit še (še means barleycorn) is $\frac{1}{6}$ of a finger, and 1 finger is $\frac{1}{12}$ of a degree, hence

$$1 \text{ še} = 1 \text{ barleycorn} = \tfrac{1}{6} \text{ finger} = \tfrac{1}{72} \text{ degree.}$$

Col. Ψ: Eclipse magnitude, in fingers.

Col. F: Daily motion of the Moon, in degrees per day.

Col. G: Duration of the preceding month. The duration is always 29 days plus a fraction of a day. The full days are left out, and the fraction is expressed in 'large hours'.

Col. J: Correction, to be subtracted from G.

Col. K: Difference in time of sunset from the day of the preceding New Moon (or Full Moon) to the present day.

Col. L: Corrected duration of the month, calculated by means of the formula

$$L = G - J + K.$$

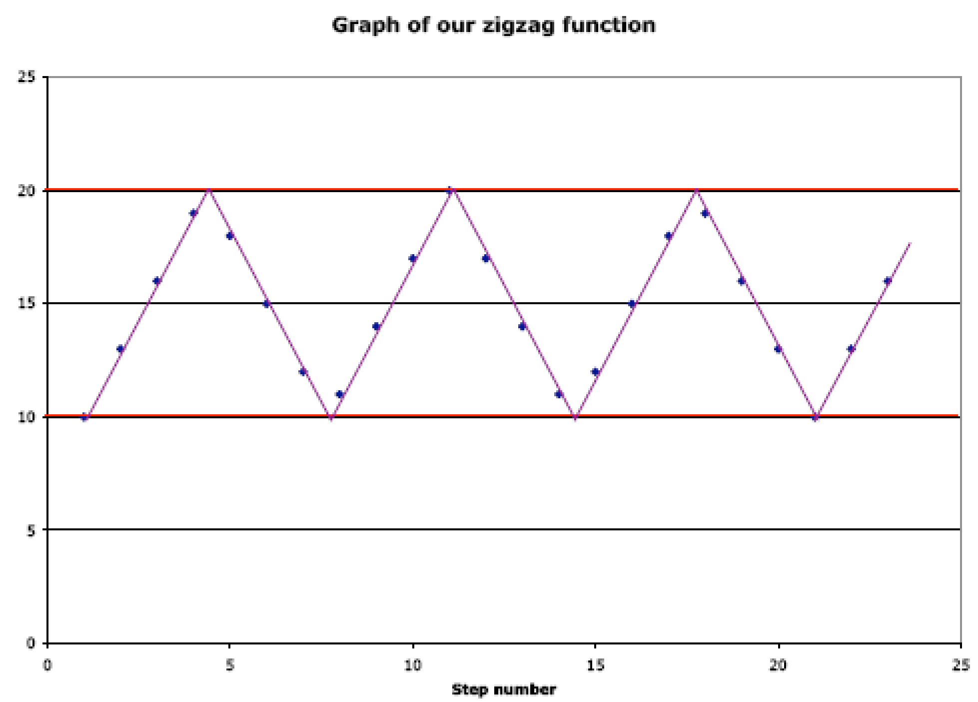
Col. M: Date and time of New Moon or Full Moon, the time being reckoned from sunset and expressed in 'large hours'.

Some texts contain two more columns P_1 and P_3. Their meaning is

P_1 = Time form sunset to the setting of the moon on the evening of first visibility of the crescent.

P_3 = Time from rising of the moon to sunrise on the morning of last visibility of the moon before New Moon.

Almost all of these changes vary fairly smoothly somewhat like sine and cosine. The Babylonian astronomers invented schemes for approximating this kind of variation.



Nothing survives to tell us how these schemes were created. What do survive are a small number of 'procedure texts' which give the rules the scribes need to compute each column.

Obv. I b

[2]lu-ma[š . . . . ta 13 zib]
[3]en 27 absin$_0$ . . [. . .] . .
[4]28,7,30 *šá al* 13 z[ib]
[5]dirig a-rá 1,4 DU ki
[6]13 zib tab ta 27 absin
[7]en 13 zib 30 tab *šá al*
[8]27 absin$_0$ dirig a-rá 56,15
[9]DU ki 27 absin tab

What follows is easy to interpret. We have two parallel rules, the first of which refers to the fast arc of the ecliptic, the second to the slow arc.

(a) From 13 ♓ to 27 ♍ month by month (you shall add) 28,7,30;
anything beyond 13 ♓ multiply by 1,4 (and) add it to 13 ♓.

(b) From 27 ♍ to 13 ♓ you shall add 30;
anything beyond 27 ♍ multiply by 56,15 (and) add it to 27 ♍.

The fast arc begins at ♓ 13 and ends at ♍ 27. Suppose that a full moon falls shortly before this arc, *e.g.*, ♓ 7 (*cf.* ephemeris No. 1 obv. III,5). According to rule (a), the next position will be found as follows: we add to ♓ 7 the arc 28;7,30 which brings us to ♈ 5;7,30. This point lies already inside the fast arc, namely 22;7,30° beyond ♓ 13. If we multiply this amount by 1;4 we obtain 23;36. This is the arc we must add to ♓ 13 in order to obtain the next position ♈ 6;36 (No. 1 obv. III,6).

Lecture 4

- *Almagest* Books 7–8
- the stars
- precession
- the constellations
- rising and setting and the calendar
- the background