Ancient Astronomy

Lecture 2

Course website: **:** www.scs.fsu.edu/~dduke/lectures

Lecture 2

- *Almagest* Book 3
- the length of the year
- the length of the seasons
- the geometric models
- the length of the day
- the background
- lost episodes in solar history

- Book 3 of the *Almagest* is about the Sun.
- The Sun is first in Ptolemy's logical structure, followed by the Moon, then the fixed stars, and finally the wandering stars (planets).
- Ptolemy says he is following Hipparchus' theory of the Sun (a claim confirmed by Theon of Smyrna).
- Probably nothing in Book 3 is original with Ptolemy, apart from the four equinox and solstice 'observations' in *Almagest* 3.1 and 3.7.

What are the major questions to be answered?

1. What is a ‘year’?

2. Is the length of the year constant?

3. What is the length of the year?

4. What are the lengths of the seasons?

5. How does the speed of the Sun vary throughout the year?

6. What kind of geometrical model would account for the phenomena (observations)?

7. How does the length of the day vary?

1. What is a ‘year’?

There are several choices:

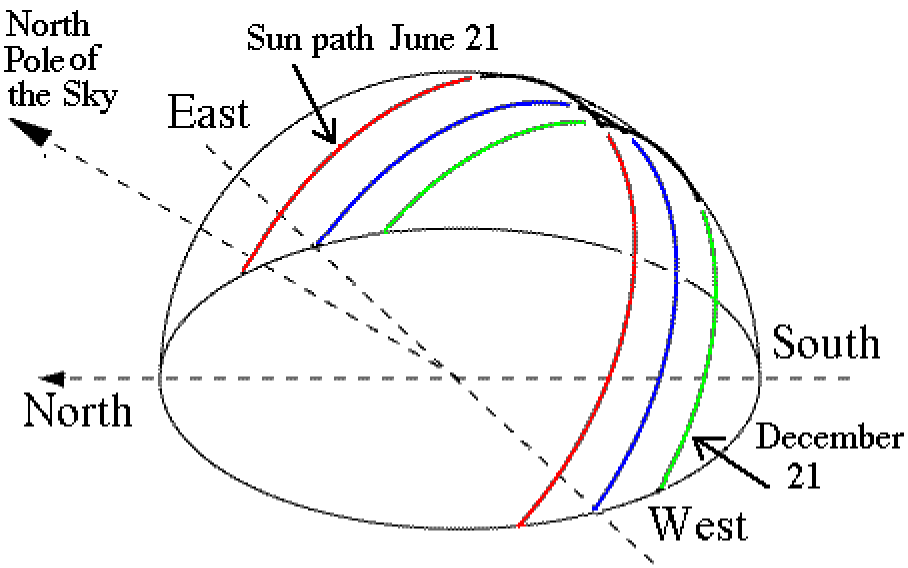
(a) return to the same star on the ecliptic (sidereal year).

(b) return to the same declination δ (e.g. the same place on the equator).

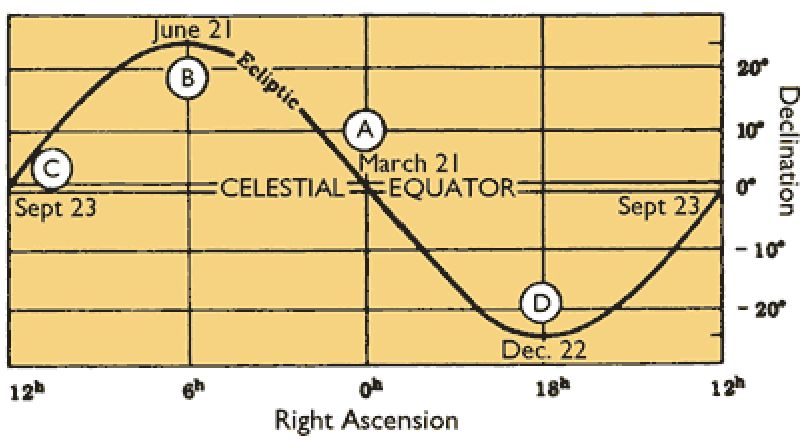
(c) return to the same speed (anomalistic).

(d) return to the same latitude (distance from the ecliptic).

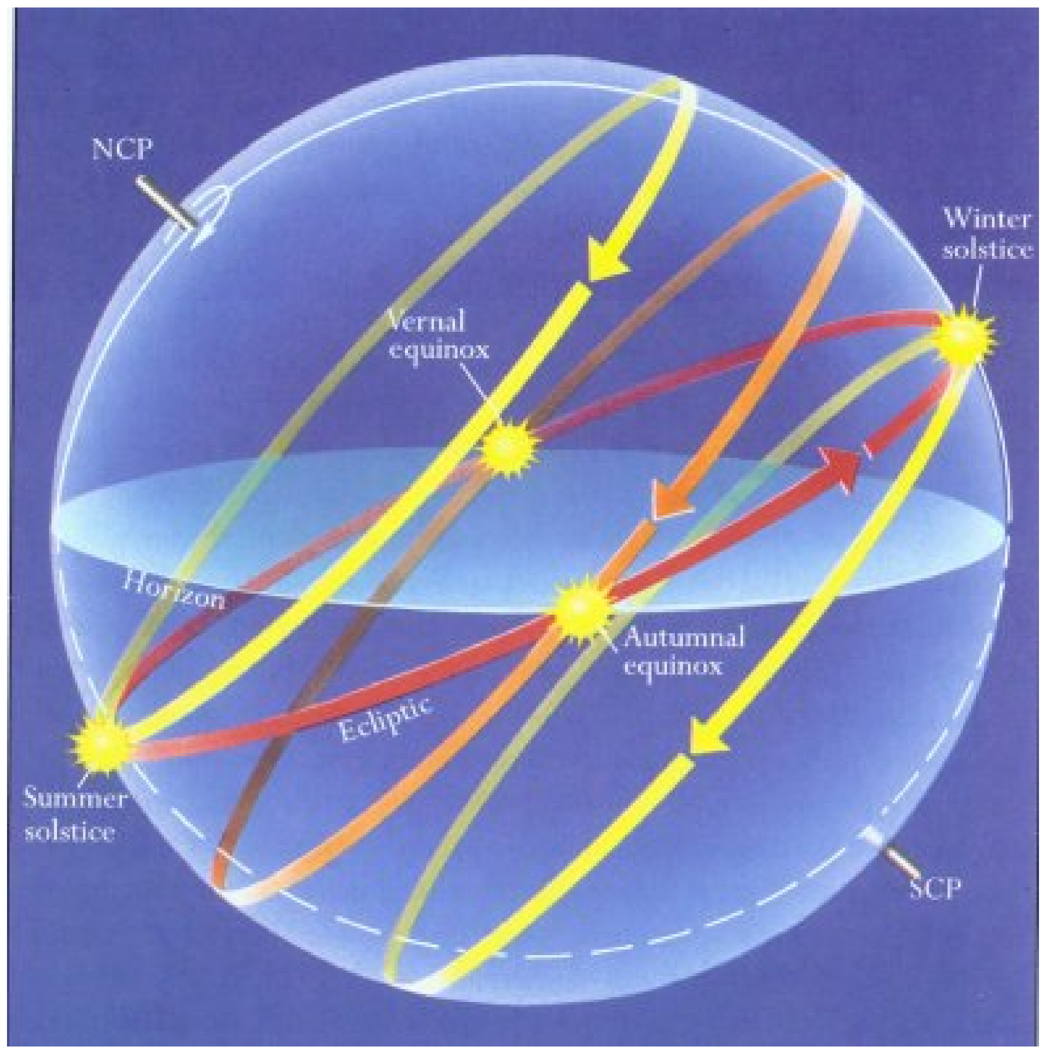
Ptolemy, and probably Hipparchus before him, chose option (b), usually called the tropical year, since you could define it as the time it takes for the Sun to return to a tropic circle, i.e. a solstice (summer or winter). Ptolemy actually measures relative to the vernal (spring) equinox.



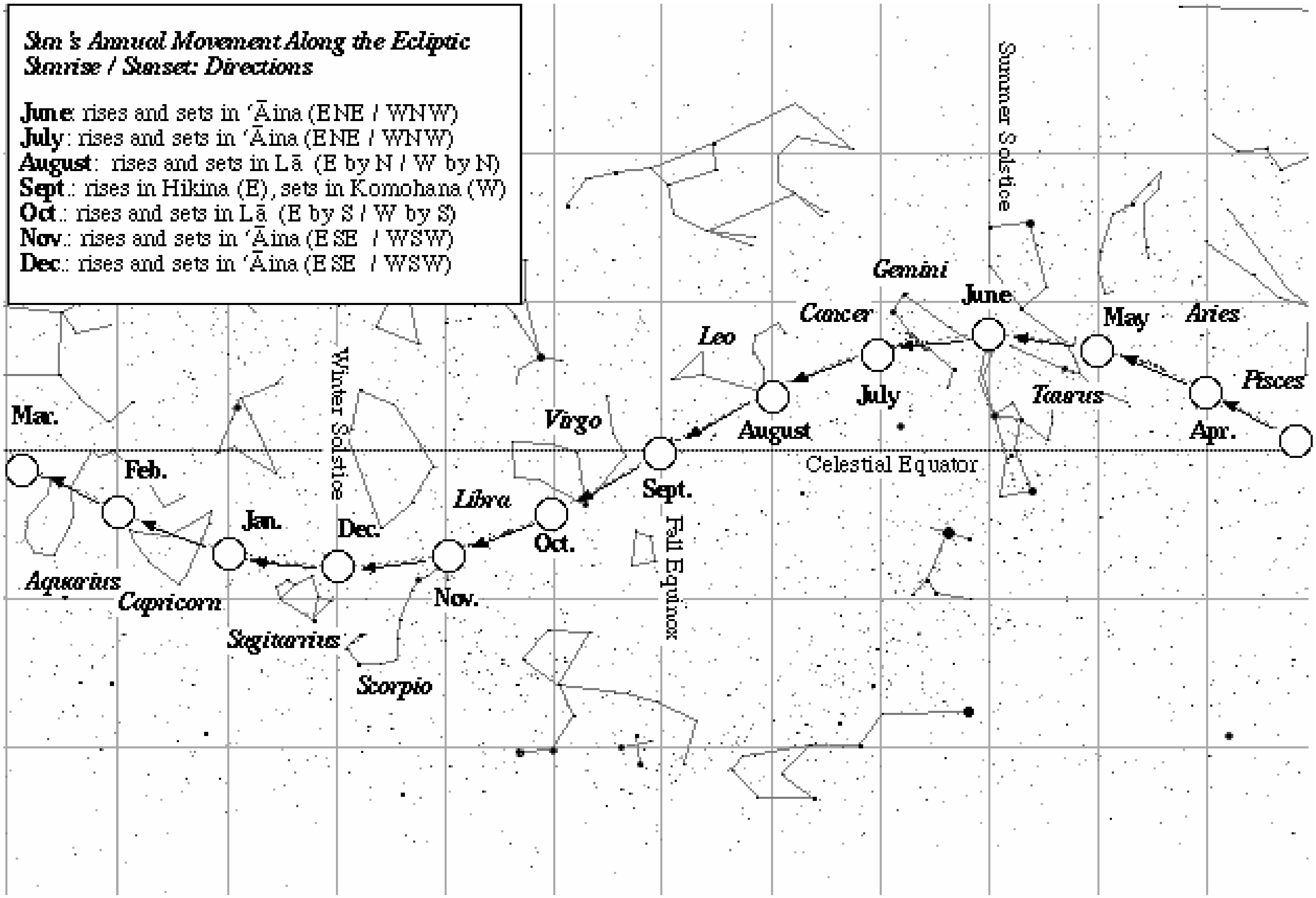
North Pole of the Sky
Sun path June 21
East
South
North
December 21
West



June 21
Ecliptic
B
A
C
D
March 21
CELESTIAL
EQUATOR
Sept 23
Sept 23
Dec. 22
20°
10°
0°
- 10°
- 20°
Declination
12h
6h
0h
18h
12h
Right Ascension



NCP
Winter solstice
Vernal equinox
Horizon
Autumnal equinox
Ecliptic
Summer solstice
SCP


Sun's Annual Movement Along the Ecliptic
Sunrise / Sunset: Directions
June: rises and sets in ʻĀina (ENE / WNW)
July: rises and sets in ʻĀina (ENE / WNW)
August: rises and sets in Lā (E by N / W by N)
Sept.: rises in Hikina (E), sets in Komohana (W)
Oct.: rises and sets in Lā (E by S / W by S)
Nov.: rises and sets in ʻĀina (ESE / WSW)
Dec.: rises and sets in ʻĀina (ESE / WSW)
Summer Solstice
Gemini
Cancer
June
May
Aries
Leo
Pisces
July
Taurus
Virgo
August
Apr.
Mar.
Celestial Equator
Feb.
Sept.
Libra
Oct.
Winter Solstice
Jan.
Dec.
Fall Equinox
Aquarius
Capricorn
Nov.
Sagittarius
Scorpio

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2–3. Is the length of the year constant, and how long?

Ptolemy says that Hipparchus measured the number of days between successive equinoxes, first autumn:

Then he lists, first, the dates of those autumn equinoxes which have been very accurately observed. One fell in the year 17 of the Third Callippic Period, Mesore 30, at the setting of the sun; and another three years after, year 20, on the first of the intercalated days in the morning, which should have been noon, so that there was a disagreement of a quarter day. And another a year after, year 21, at the sixth hour, which agreed with the preceding observation. And another eleven years after, year 32, on the third of the intercalated days at midnight before the fourth. And it should have been in the morning, so that there was again a disagreement of a quarter day. And another a year after, year 33, on the fourth of the intercalated days in the morning, which agreed with the preceding observation. And another three years after, year 36, on the fourth of the intercalated days in the evening. And it should have been at midnight so that there was again a disagreement of a quarter day.

then spring:

And next he lists those spring equinoxes likewise accurately observed. One fell in the year 32 of the Third Callippic Period, Mechir 27, in the morning. But he adds: "The ring in Alexandria was also lighted up equally on both sides at the fifth hour, so that the same equinox differently observed differed by near-nearly five hours." And he says the equinoxes following, up to the year 37, agreed with the addition of a quarter day. And eleven years after, year 43, Mechir 29-30, just after midnight, he says, there was a spring equinox, which also agreed with the observation in the year 32; and also, he says, with the observations in the following years up to the year 50. For in that year it fell on Phamenoth 1 at sunset, within very nearly 1¾ days of that of the year 43, which is also proportional to the 7 intervening years. And so in these observations there was no perceptible difference although it is possible for there to be an error of as much as a quarter day, not only as regards the tropic observations, but also the equinoctial. For even if the position or discrimination of the instru-

so Hipparchus finds that with a few exceptions the year is 365 ¼ days. Further, the exceptions could easily be observation uncertainties, so Ptolemy finds no reason to doubt that the year length is constant.

Getting a 'precise' year length.

Ptolemy says that Hipparchus found a year length of $365+\frac{1}{4}-\frac{1}{300}$ days (probably from the interval between the summer solstices in 280 B.C. and 135 B.C.)
Ptolemy then says

have used for this comparison the observation of the equinoxes, and, because of their accuracy, especially those given Hipparchus' approval as having been most certainly taken by him, and those most carefully observed by ourselves with the instruments for such purposes described at the beginning of this treatise. And from these we find that, in very nearly 300 years, the tropics and equinoxes fall one day sooner than the quarter-day addition to 365 days allows.

Ptolemy gives the dates for the autumn equinoxes of 132 and 139 and the spring equinox and summer solstice of 140, all “most carefully observed”, and compares them to the fall equinox of -146, the spring equinox of -145, and the summer solstice of -431:

(1) -146/9/27 midnight to 132/9/25 2 pm
(2) -146/9/27 midnight to 139/9/26 7 am
(3) -145/3/24 6 am to 140/3/22 1 pm
(4) -431/6/27 6 am to 140/6/25 2 am

in each case you count the number of intervening days, divide by the number of years, and the year length is $365+\frac{1}{4}-\frac{1}{300}$ days.

The correct value is about $365+\frac{1}{4}-\frac{1}{133}$, about 6 minutes shorter.

What are the major questions to be answered?

1. What is a ‘year’?

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4. What are the lengths of the seasons?

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6. What kind of geometrical model would account for the phenomena (observations)?

7. How does the length of the day vary?

4–5. the length of the seasons and the varying speed of the Sun

Early History of Season Lengths

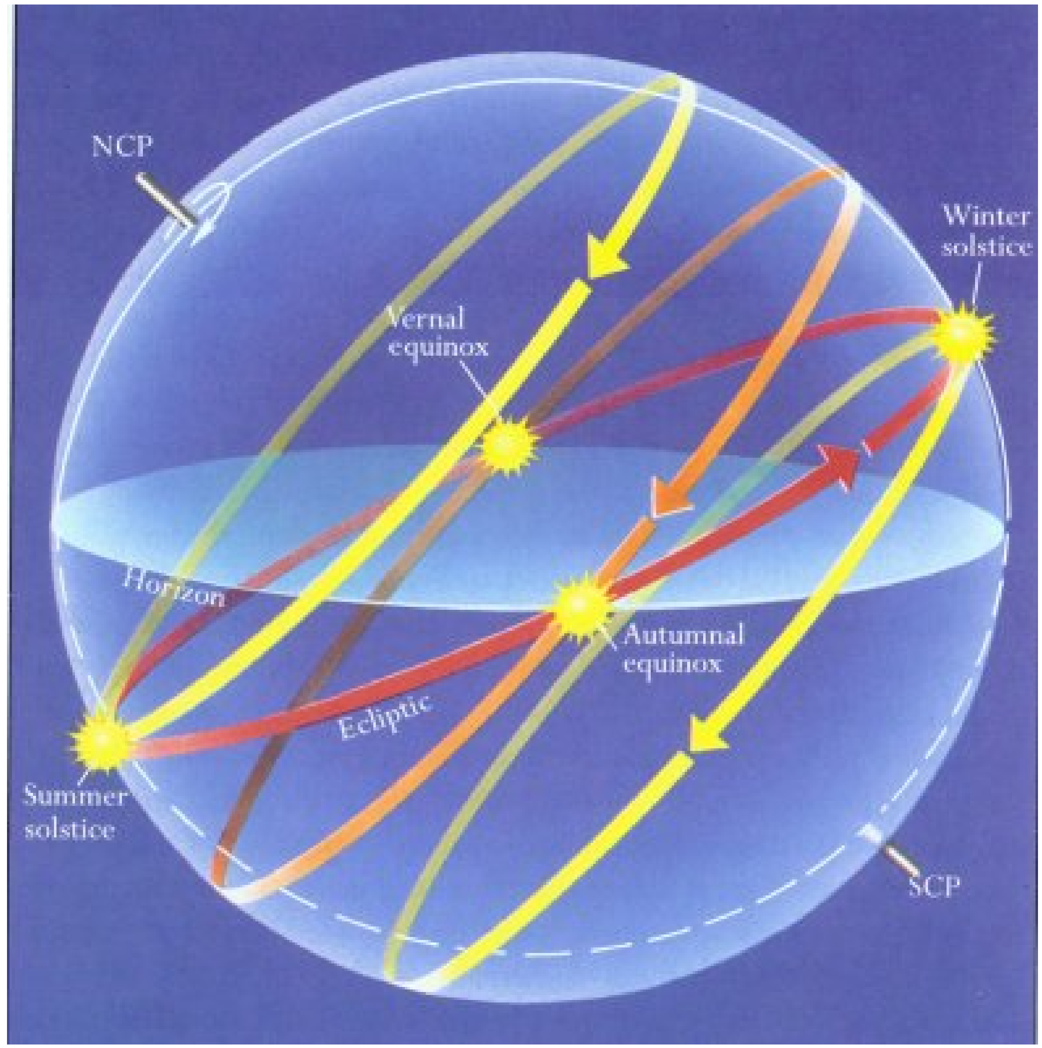
based on a 365 day calendar (except for Hipparchus)

	Summer	Autumn	Winter	Spring
Democritus (460 B.C.)	91	91	91	92
Euctomen (432 B.C.)	90	90	92	93
Eudoxus (380 B.C.)	91	92	91	91
Callippus (340 B.C.)	92	89	90	94
Geminus (200 B.C.)	92	89	89	95
Hipparchus (130 B.C.)	92½	88⅛	90⅛	94½
accurate (134 B.C.)	92⅓	88⅔	90¼	94

other than Hipparchus, it is not at all certain that any of these were based on observation of equinoxes or solstices.

Ptolemy says that Hipparchus assumed season lengths 94½ days for Spring and 92½ days for Summer, but he does not say how Hipparchus got these values.

This tells us that the Sun does not appear to move around the ecliptic at a uniform speed.



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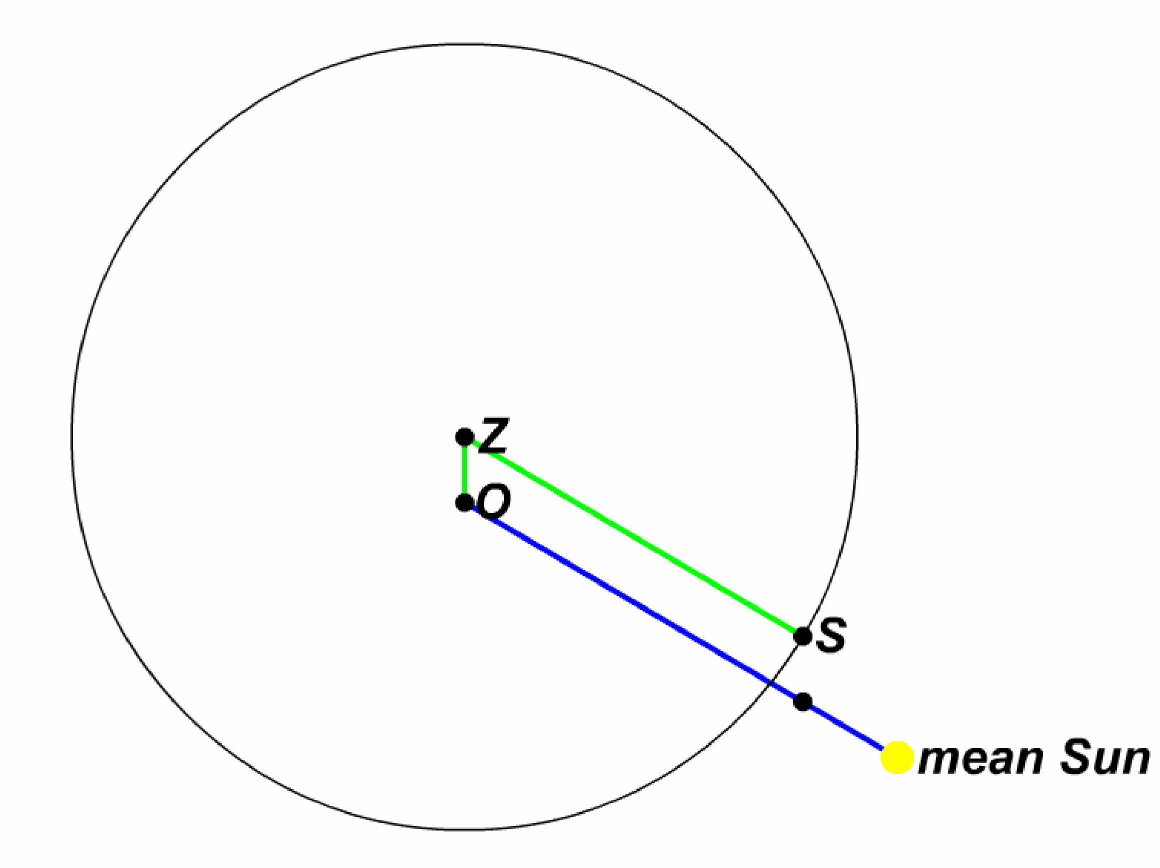

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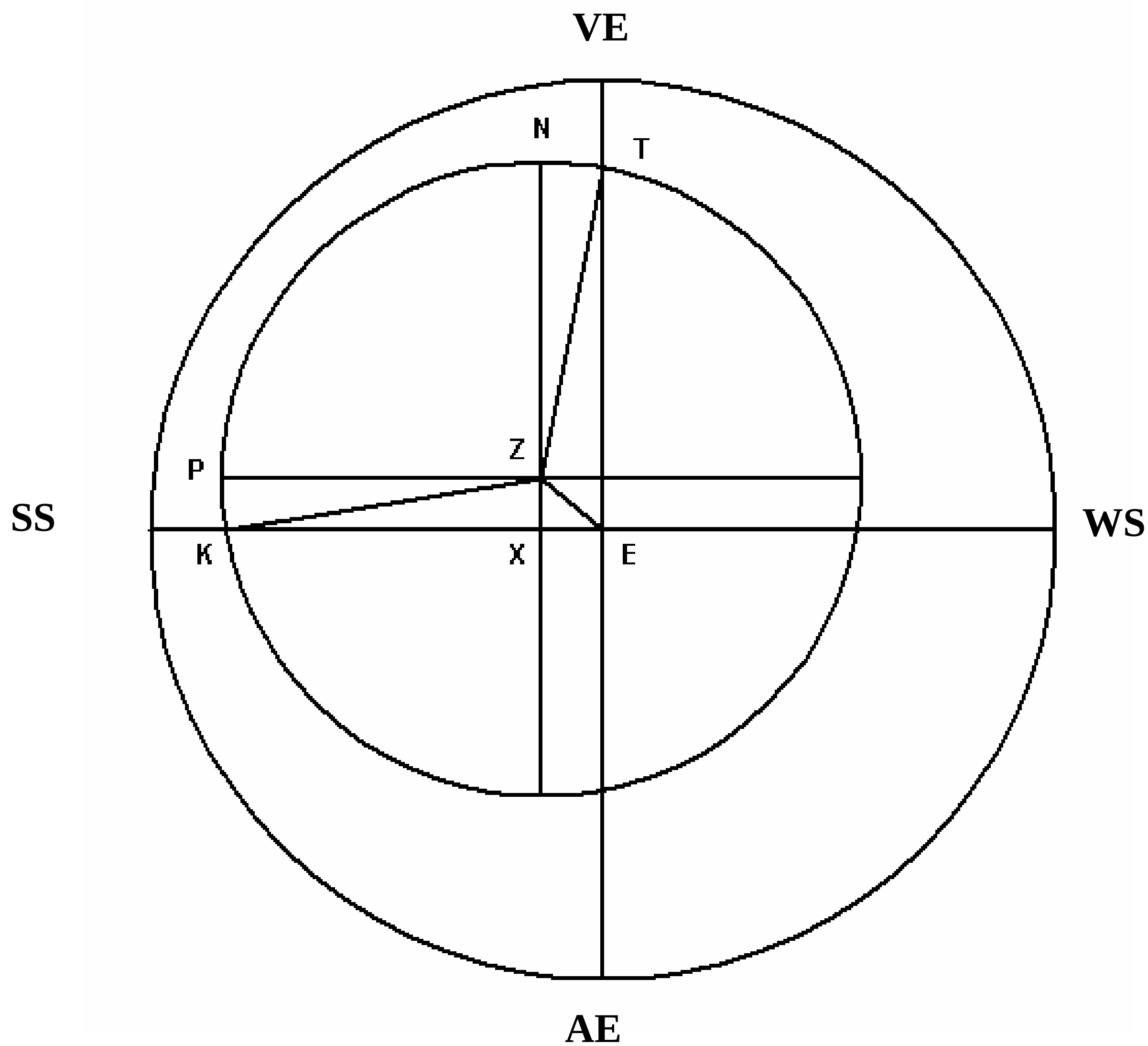
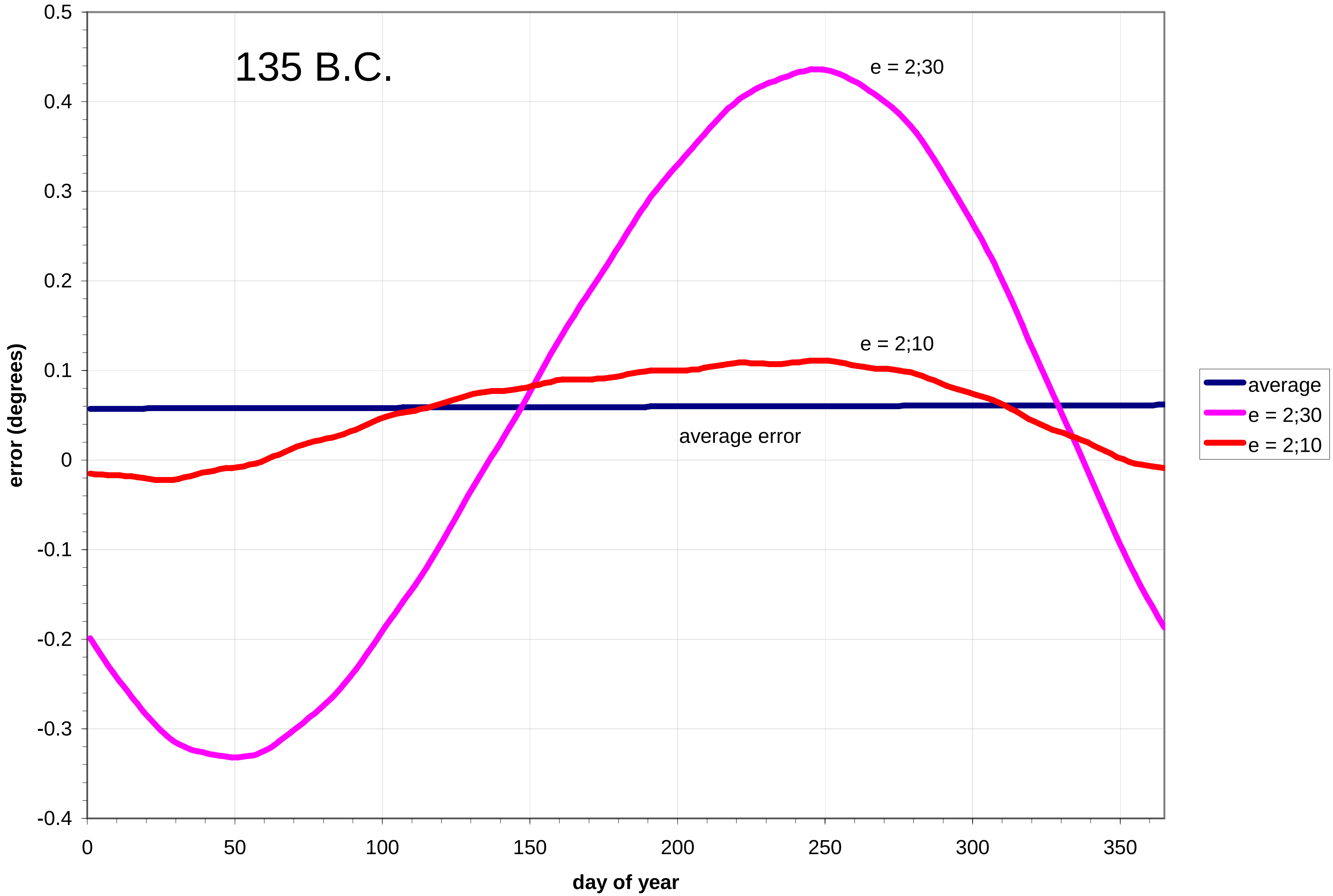
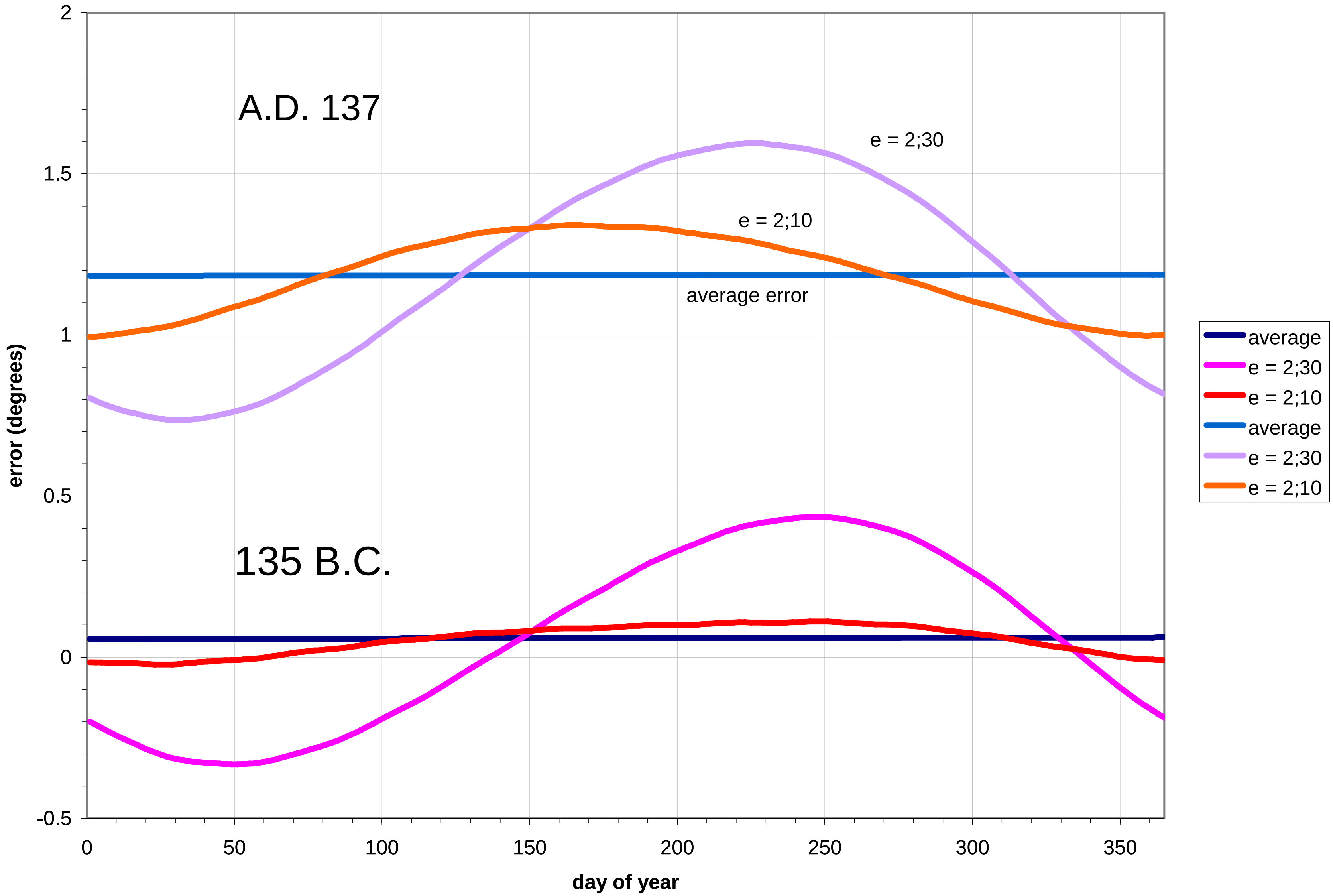
from the season lengths we know angles *TZN* and *PZK*, and using simple geometry gives *EZ* = *e* = 2;30 and angle *TEZ* = 65;30°.



How accurate is the model? Not bad in Hipparchus' time….



much less so in Ptolemy's time (large average error).



Some key terms:

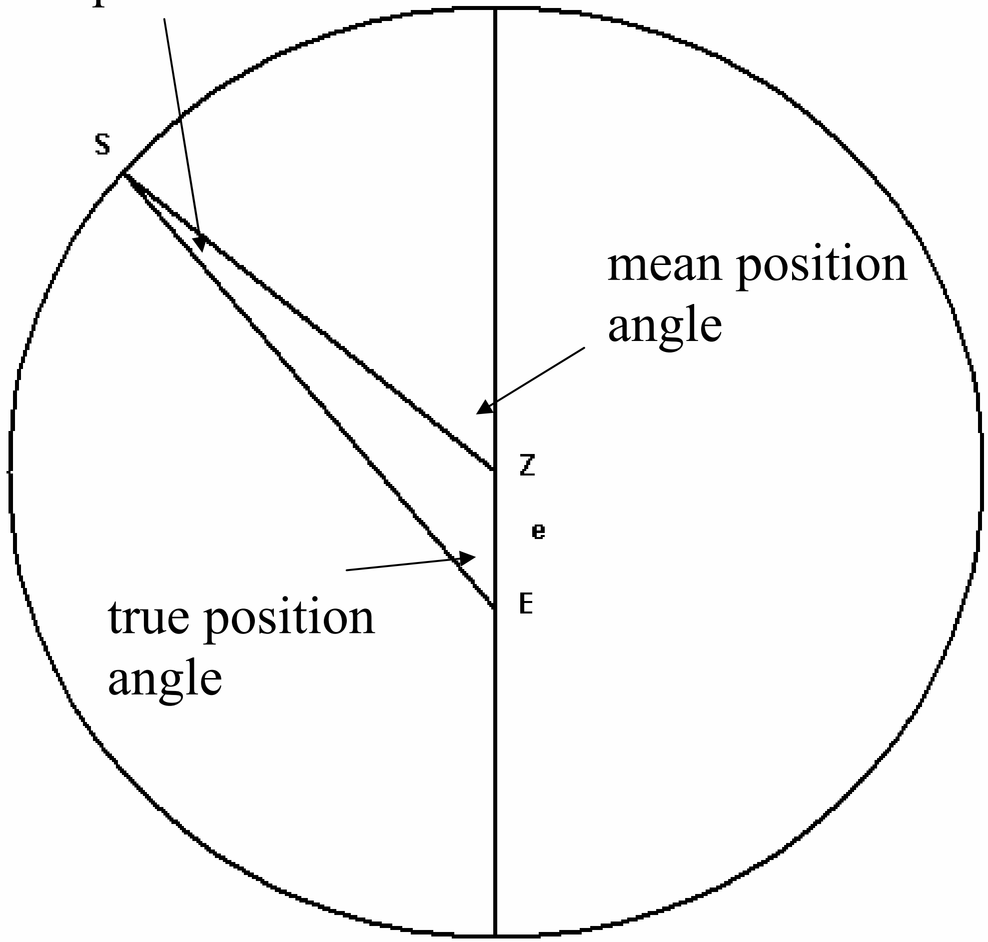
mean motion refers to the *average* speed of some celestial body. They knew that the speed could vary around the orbit, but they knew the average speed was the distance around many orbits divided by the time for many orbits. Mean motion is *regular*.

mean position refers to where the body *would* be if it always traveled with *mean speed*. In reality the true position of the body would usually be ahead of or behind the mean position.

motion in anomaly is the regular motion that actually causes the true motion to differ from the mean motion, so the true motion appears to be irregular. Thus *irregular* motions result from a compounding of *regular motions* (mean and anomaly).

the *equation* is an angle that is the difference between the true position and the mean position of a body. Thus

true = mean + equation



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the length of the day is determined mostly by how fast the Earth rotates about its axis (or the celestial sphere, to the ancients).

However, because

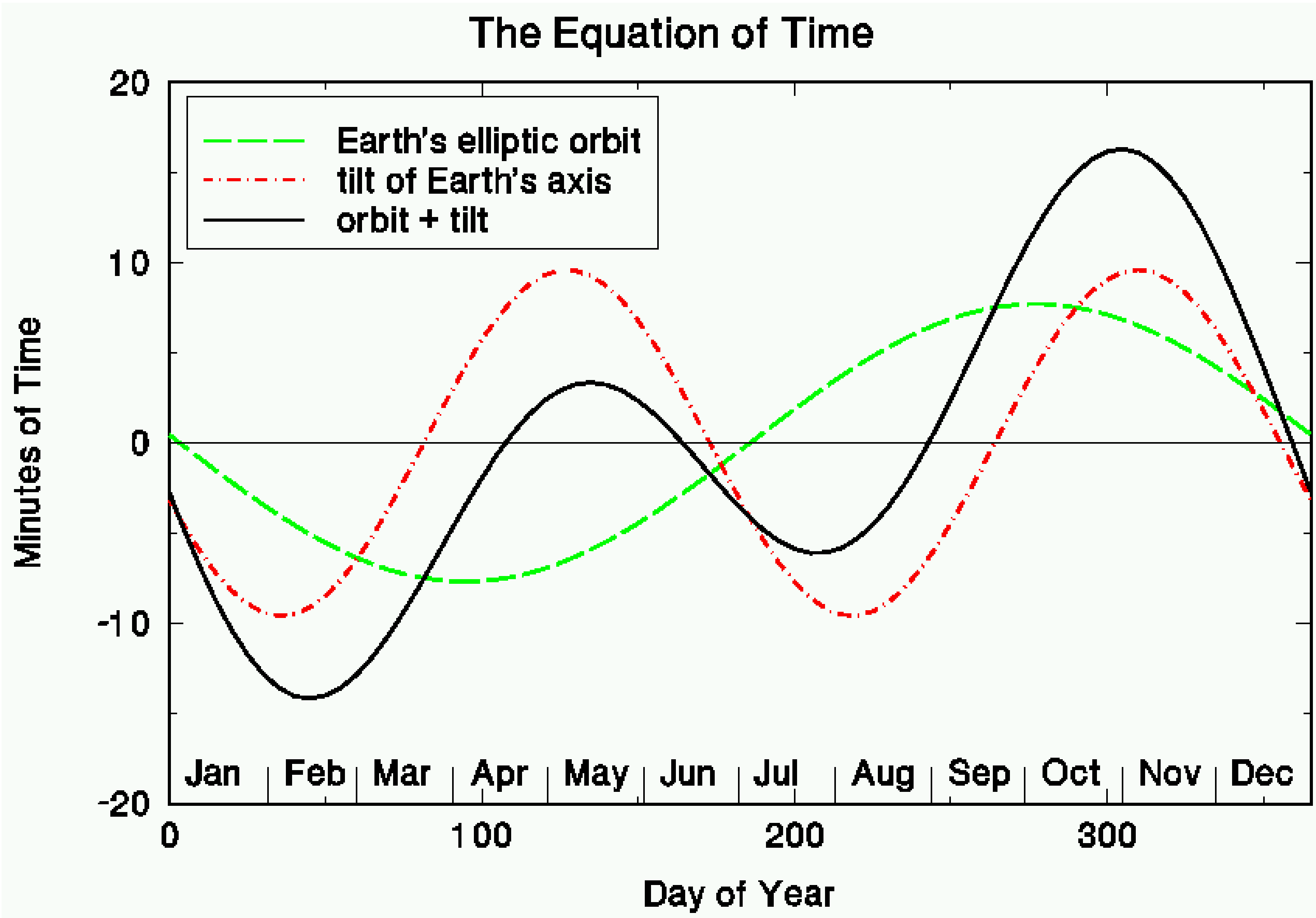
(b) the Sun is moving on the oblique ecliptic, and

(c) the speed of the Sun varies on the ecliptic,

the actual length of time between successive noon's varies slightly.

From day to day the variation is very small, but it does accumulate so that a day in February can be about 15 minutes shorter than average, while a day in November can be about 15 minutes longer than average, etc.

Ptolemy understood this very well, but does not tell us how he learned it.



Main features of Hipparchus' solar model as reported by Ptolemy:

- there is only *one* variation: the speed around the ecliptic.
- the eccentric and epicycle versions give equivalent explanations.
- apogee is the direction of slowest motion on the ecliptic, perigee is the direction of fastest motion.
- the direction of the apogee is always 65½° from the vernal equinox and the eccentricity is always 2;30 (compared to 60). Both are determined from the season lengths for Spring and Summer (94½ and 92½ days).
- Ptolemy insists the model must predict that the time from slowest to mean (average) speed is *greater* than the time from mean to fastest speed, "for we find that this accords with the phenomena [observations]".
- the Sun is always on the ecliptic, never north or south (no latitude).

The Background

A closer look at Ptolemy's 'most carefully observed' equinox and solstice dates.

(1) -146/9/27 midnight to 132/9/25 2 pm
(2) -146/9/27 midnight to 139/9/26 7 am
(3) -145/3/24 6 am to 140/3/22 1 pm
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suppose we *compute* the expected date of each event by multiplying the number of intervening years by the *assumed* days per year, $365+\frac{1}{4}-\frac{1}{300}$. We get

	correct
(1) 132/9/25 1:46 pm	(9/24 4 am)
(2) 139/9/26 7:12 am	(9/24 9 pm)
(3) 140/3/22 1:12 pm	(3/21 4 pm)
(4) 140/6/25 2:19 pm	(6/23 1 am)

Was it just too hard for Ptolemy to get right?

-161/9/27 6 pm	(9/27 2 am)	-141/3/24 6 am	(3/24 2 pm)
-158/9/27 6 am	(9/26 8 pm)	-140/3/23 noon	(3/23 8 pm)
-157/9/27 noon	(9/27 2 am)	-134/3/24 midnight	(3/24 7 am)
-146/9/27 midnight	(9/26 6 pm)	-133/3/24 6 am	(3/24 1 pm)
-145/3/24 6 am	(3/24 3 pm)	-132/3/23 noon	(3/23 6 pm)
-145/9/27 6 am	(9/26 11pm)	-131/3/23 6 pm	(3/24 midnight
-144/3/23 noon	(3/23 9 pm)	-130/3/24 midnight	(3/24 6 am)
-143/3/23 6 pm	(3/24 2 am)	-129/3/24 6 am	(3/24 noon)
-142/3/24 midnight	(3/24 8 am)	-128/3/23 noon	(3/23 6 pm)
-142/9/26 6 pm	(9/26 5 pm)	-127/3/23 6 pm	(3/23 11pm)

so apparently Hipparchus was generally accurate to the nearest ¼ day.

Clearly Ptolemy *computed* those four dates. *Why* did he do that? There is no objective evidence to help us, so we can only speculate. What we *can* say with some certainty is that this was the rule, not the exception, for Ptolemy.

Ptolemy tells us that his solar model is the same as Hipparchus' but gives us no other background information. In Book 12 he does mention that Apollonius of Perge (*ca.* 200 B.C.) had proved a rather complicated theorem involving the epicycle model, so it seems likely that epicycles and eccentrics, and their equivalence, had been studied for several centuries.

Also, as we shall see for the Moon and the planets, those models require both moving apogee directions and (effectively) oscillating eccentricities. One might think that for uniformity and unity Ptolemy would make the solar model more like the models for the Moon and the planets, but he does not.

What Ptolemy does *not* tell us is that there was a lot of other activity developing solar models both near his time and going back centuries.

Here are some examples:

(1) There are clear indications in the *Almagest* that Hipparchus himself used solar models different from the standard one attributed to him by Theon and Ptolemy. For example, the two pairs of eclipse longitude differences that Hipparchus uses to find the unusual lunar eccentricities in *Almagest* 4.11 may also be used to deduce the underlying solar models, and the resulting parameters are equally unusual: e=7;48 and A=76;25° for Trio A, and e=3;11 and A=46;09° for Trio B. Although attempts have been made to understand the underlying models, the analyses are neither conclusive nor satisfying. The solar parameters are so bizarre that we might be tempted to speculate that Hipparchus is somehow trying to use a lunar theory to learn something about the time variation of solar theory (the trios date to about –380 and –200), and so it is perhaps interesting that in both trio analyses the eclipses all occur near equinoxes and solstices [more about this case in Lecture 3].

(2) *Almagest* 5.3 and 5.5 give three timed solar longitudes due to Hipparchus, and these imply a solar model with parameters $e = 2;16$ and $A = 69;05°$, although it might be that the underlying model is actually based on season lengths of 94¼ days and 92½ days, for which the parameters are instead $e = 2;19$ and $A = 67;08°$. Either way, the value of e is significantly improved over the 'standard' Hipparchan value 2;30.

(3) Theon of Smyrna mentions, quite matter-of-factly, a solar model with periods of 365¼ days in longitude, 365½ days in anomaly, and 365⅛ days in latitude. He also mentions that the Sun strays from the ecliptic by ±½°. Solar latitude was mentioned as early as Eudoxus, and must have had some level of use, since not only Theon but also Pliny mentions it, and Hipparchus felt compelled to deny its existence. Ptolemy never mentions solar latitude.

(4) *P. Oxy LXI.4163* is a fragment of a papyrus table from Oxyrhynchus that gives a template for daily longitudes of the Sun to degrees and minutes starting from the day of summer solstice. All indications are that it is not based on the usual Hipparchan parameters.
(5) *P. Oxy LIX.4162* is similar to *P. Oxy LXI.4163* but appears to count days starting when the Sun is at perigee and puts the cardinal points at 8° of the signs. In this case the indications are strong that the underlying theory is kinematical, but even if it is, it seems not likely to be based on the usual Hipparchan parameters.
(6) *P. Oxy. LXI.4148* is a table of dates of summer solstices over a series of years. The dates are in error by about five days in the years covered in the fragment and are based on a year of length 365;15,22,46 days. There are indications that the dates might have begun from a known Hipparchan summer solstice measurement of –127 June 26 at sunrise.

For more information on the astronomical papyri of Oxyrhynchus see
http://www.chass.utoronto.ca/~ajones/oxy/

We know from the *Almagest* that Hipparchus knew the times of equinoxes and summer solstices to an average accuracy of about ¼ day. Since no ancient source explains how these times were determined, we need to consider just how an ancient astronomer would measure the time of an equinox or a solstice to that level of accuracy.

By definition,

- an equinox occurs at the moment the Sun touches the equator, so its declination $\delta = 0°$.

- and a solstice occurs when the Sun touches either tropic circle (Cancer to the north, Capricorn to the south), so its declination $\delta = \varepsilon = 23;43^\circ$

It is clear from practical considerations that no one could have reliably and routinely simply noted the moment when the Sun's declination was at a given value: 0° for an equinox or ±23;43° for a solstice.

- On the one hand, about half of the events will occur at night, when the Sun is not visible.

- On the other hand, even if the event happens in daylight, it is not always the case that the Sun will be unobscured by clouds and in a position in the sky favorable for measuring the declination accurately.

In addition, for the solstices it is impossible to achieve ¼ day accuracy with naked eye observations of any kind within a day or so of the event since the declination of the Sun is changing extremely slowly near a solstice.

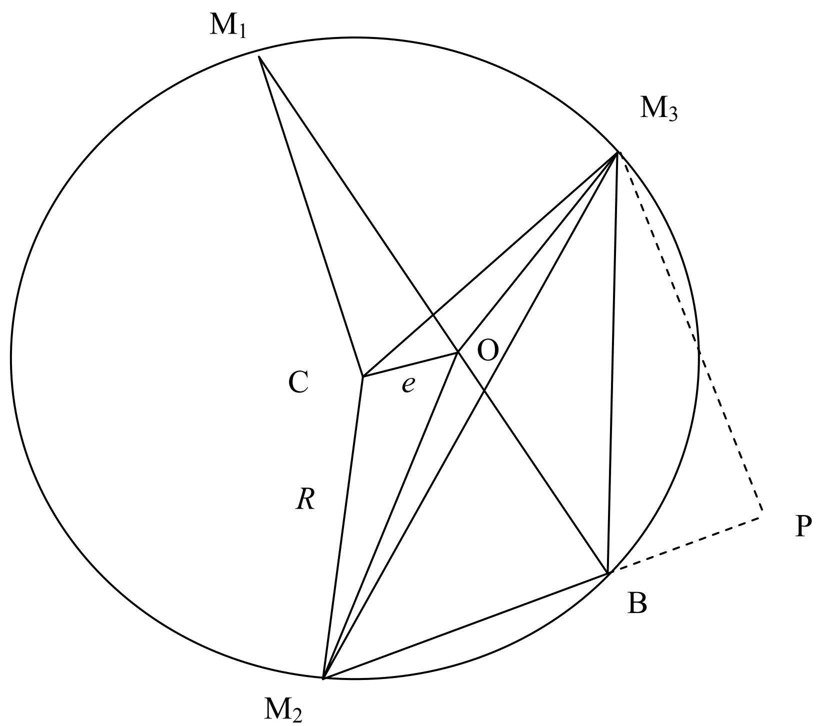
It is most likely, then, that equinoxes and solstices were determined by observing noon solar altitudes for *a series of days before and after the events*.

When the Sun is crossing the meridian at noon, it is relatively easy to measure its altitude, and then knowing the geographical latitude, to compute the declination. From the declination, it is easy to compute the Sun's position on the ecliptic (the longitude), and we know that Hipparchus knew how to do it.

But it is only at noon that such an easy determination is possible. It is then fairly straightforward to estimate the time that the Sun's declination reaches some specific targeted value: 0° for an equinox, and maximum or minimum for a solstice.

That series of daily altitude measurements were used to determine the time of cardinal events can hardly be doubted, even though no surviving ancient source has documented such an episode. Especially for the solstices, it is essentially the only viable option for achieving ¼ day accuracy.

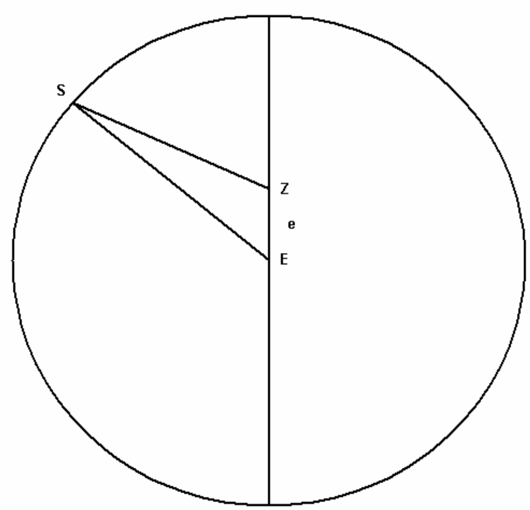
In fact, however, you don't really need equinoxes or solstices. Any trio of timed longitudes would be adequate. Ptolemy provides two such analyses for the Moon and one each for Mars, Jupiter and Saturn. Hipparchus, and perhaps his predecessors and certainly his successors, knew the method, so it seems inconceivable that it was not used multiple times to also determine solar model parameters.



Finally, Ptolemy insists the model must predict that the time from slowest to mean (average) speed is *greater* than the time from mean to fastest speed, “for we find that this accords with the phenomena [observations]”.

First, the time differences Ptolemy refers to are undetectable using naked eye observations, so this is something he is pushing not from empirical observations, but from some unstated theoretical (or philosophical) bias.

Second, there is a perfectly good model for the solar motion that violates Ptolemy’s rule: the concentric equant. Using the concentric equant one finds that the time from least speed to mean speed is *equal* to the time from mean speed to greatest speed.



In the concentric equant model the Earth is at the center E of the deferent, but the center of uniform motion Z of the Sun S is displaced some distance e from the center. Even though the Sun is now always at the same distance R = ES from the Earth, the model still produces an apparent speed variation in the motion of the Sun such that in one direction (the direction EZ) the Sun seems to be moving slowest, and in the opposite direction it seems to be moving fastest.

The concentric equant model for the Sun is repeatedly attested in Indian texts, all of which are generally supposed to be of Greco-Roman origin, and the accurate value $e = 2;10$ is routinely used.

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For the equation of time, remember that it has two causes:

(a) the Sun is moving on the oblique ecliptic, and
(b) the speed of the Sun varies on the ecliptic,

so the actual length of time between successive noon's varies slightly.

It turns out that Geminus, writing in about 50 B.C., mentions the equation of time but for him only (a) is involved.

In the ancient Indian texts, on the other hand, the equation of time is attributed only to (b). These texts are supposed to originate from Greco-Roman sources from the time period between about 100 B.C. and A.D. 100, or post-Hipparchus and pre-Ptolemy.

Early History of Season Lengths

based on a 365 day calendar (except for Hipparchus)

	Summer	Autumn	Winter	Spring
Democritus (460 B.C.)	91	91	91	92
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other than Hipparchus, it is not at all certain that any of these were based on observation of equinoxes or solstices.

Lecture 3

- *Almagest* Books 4–5.10
- the Moon
- the problem of parallax
- the length of the various months
- the first geometric model
- the second geometric model
- the background