Ancient Astronomy

Lecture 1

Course website: www.scs.fsu.edu/~dduke/lectures

Lecture 1

- Where, When and Who
- *Almagest* Books 1 and 2
- the celestial sphere
- numbers and angles (sexagesimal base-60)
- obliquity and latitude and the related instruments
- plane geometry and trigonometry, the chord tables
- spherical trigonometry, circles on the celestial sphere

the Where: Ptolemy's World A.D. 150

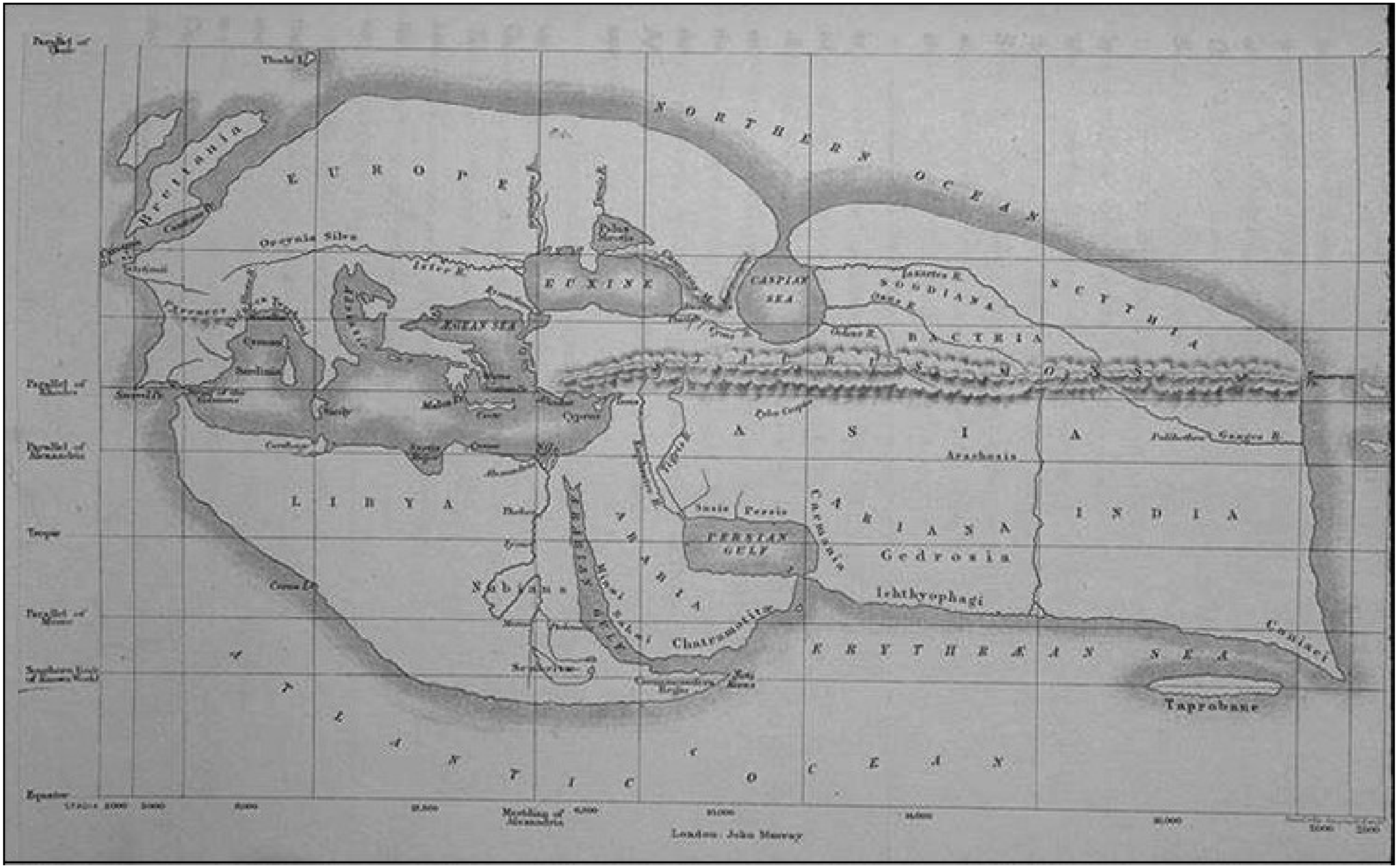
The world as our story begins. The East


Greek Colonies, c. 550 B.C.E.
N
Massilia
CORSICA
Alalia
ITALY
Adriatic Sea
Black Sea
Zacynthus
Olbia
Epidamnus
Byzantium
Balearic Islands
Cumae
Tarantum
Sybaris
Croton
Athens
LYDIA
SARDINA
Locri
Corinth
Miletus
Catania
Sparta
Syracuse
Rhodes
IONIA
Mediterranean Sea
CRETE
CYPRUS
NORTH ARFICA
Cyrene
Naucratis
Area of Greek Settlement

and the West

the Greek's near their peak

Alexander the Great’s empire



Strabo's Geography (1st-2nd century B.C.)

Ptolemy’s World Map (1st century A.D.)

The Spread of Islam. The rapid spread of Islam created within a century a unified cultural and economic zone from India to the Atlantic Ocean within.

Most of what we have from antiquity was preserved and transmitted to us by the Islamic societies of the 8th – 13th centuries A.D.

Who and When: Ancient Astronomers

Homer/Hesiod	-750	Aratus	-270
Meton/Euctomen	-430	Timocharis	-260
Eudoxus	-380	Aristarchus	-240
Aristotle	-340	Archimedes	-220
Heraclides	-330	Eratosthenes	-210
Callippus	-330	Apollonius	-200
Autolycus/Euclid	-330	Hipparchus	-130
Aristyllus	-300	Posidonius	-100
Berosus	-300	Geminus	-50

Theon of Smyrna	120
Ptolemy *Almagest*	150
Theon of Alexandria	350

Relevant Famous People

Plato	-375	philosopher
Alexander the Great	-330	conquered Babylon
Strabo	10	*Geography*
Pliny	70	*Natural History*
Plutarch	100	*Concerning Nature*
		The Face in the Moon
Marinus of Tyre	120	geography (Ptolemy's source)

Later Famous Astronomers (and Ptolemy influenced every one of them)

Hipparchus	-130
Ptolemy *Almagest*	150
Aryabhata (India)	500
al-Sufi (Islam)	950
al-Tusi/Urdi/Shatir	1250
Ulugh Beg	1420
Copernicus	1540
Tycho Brahe	1570
Kepler	1620
Newton	1680

Almagest, Book I begins:

The true philosophers, Syrus,[5] were, I think, quite right to distinguish the theoretical part of philosophy from the practical. For even if practical philosophy, before it *is* practical, turns out to be theoretical,[6] nevertheless one can see that there is a great difference between the two: in the first place, it is possible for many people to possess some of the moral virtues even without being taught, whereas it is impossible to achieve theoretical understanding of the universe without instruction; furthermore, one derives most benefit in the first case [practical philosophy] from continuous practice in actual affairs, but in the other [theoretical philosophy] from making progress in the theory. Hence we

and a bit later:

larly applied. For Aristotle divides theoretical philosophy too, very fittingly, into three primary categories, physics, mathematics and theology.[7] For everything that exists is composed of matter, form and motion; none of these [three] can be observed in its substratum by itself, without the others: they can only be imagined. Now the first cause of the first motion of the universe, if one considers it simply, can be thought of as an invisible and motionless deity; the division [of theoretical philosophy] concerned with investigating this [can be called] 'theology', since this kind of activity, somewhere up in the highest reaches of the universe, can only be imagined, and is completely separated from perceptible reality. The division [of theoretical philosophy] which investigates material and ever-moving nature, and which concerns itself with 'white', 'hot', 'sweet', 'soft' and suchlike qualities one may call 'physics'; such an order of being is situated (for the most part) amongst corruptible bodies and below the lunar sphere. That division [of theoretical philosophy] which determines the nature involved in forms and motion from place to place, and which serves to investigate shape, number, size, and place, time and suchlike, one may define as 'mathematics'. Its subject-matter falls as it were in the middle between the other two, since, firstly, it can be conceived of both with and without the aid of the senses, and, secondly, it is an attribute of all existing things without exception, both mortal and immortal: for those things which are perpetually changing in their inseparable form, it changes with them, while for eternal things which have an aethereal[8] nature, it keeps their unchanging form unchanged.

From all this we concluded:[9] that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them; and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the

> The general preliminary discussion covers the following topics: the heaven is spherical in shape, and moves as a sphere; the earth too is sensibly spherical in shape, when taken as a whole; in position it lies in the middle of the heavens very much like its centre; in size and distance it has the ratio of a point to the sphere of the fixed stars; and it has no motion from place to place. We shall briefly discuss each of these points for the sake of reminder.

similarly (see the excerpts on the supplementary reading page):
Theon of Smyrna (about A.D. 120)
Strabo *Geography* (about A.D. 5)
Geminus (about 50 B.C.)
Hipparchus (about 130 B.C.)
Autolycus (about 300 B.C.), and Euclid's *Phenomena* is similar
Eudoxus (about 320 B.C.)
Aristotle (about 350 B.C.)
Hesiod (about 750 B.C.)
Homer (about 780 B.C.)

It is fair to say that Ptolemy makes the best effort to give fairly cogent arguments, usually astronomical, to support all of these assumptions.

For example:

7. {*That the earth does not have any motion from place to place, either*}[37]

One can show by the same arguments as the preceding that the earth cannot have any motion in the aforementioned directions, or indeed ever move at all from its position at the centre. For the same phenomena would result as would if it had any position other than the central one. Hence I think it is idle to seek for causes for the motion of objects towards the centre, once it has been so clearly established from the actual phenomena that the earth occupies the middle place in the universe, and that all heavy objects are carried towards the earth. The following fact alone would most readily lead one to this notion [that all objects fall towards the centre]. In absolutely all parts of the earth, which, as we said, has been shown to be spherical and in the middle of the universe, the direction[38] and path of the motion (I mean the proper, [natural] motion) of all bodies possessing weight is always and everywhere at right angles to the rigid plane drawn tangent to the point of impact. It is clear from this fact that, if

But certain people,[41] [propounding] what they consider a more persuasive view, agree with the above, since they have no argument to bring against it, but think that there could be no evidence to oppose their view if, for instance, they supposed the heavens to remain motionless, and the earth to revolve from west to east about the same axis [as the heavens], making approximately one revolution each day;[42] or if they made both heaven and earth move by any amount whatever, provided, as we said, it is about the same axis, and in such a way as to preserve the overtaking of one by the other. However, they do not realise that, although there is perhaps nothing in the celestial phenomena which would count against that hypothesis, at least from simpler considerations, nevertheless from what would occur here on earth and in the air, one can see that such a notion is quite ridiculous. Let us concede to them [for the sake of argument] that such an unnatural thing could happen as that the most rare and light of matter should either not move at all or should move in a way no different from that of matter with the opposite nature (although things in the air, which are less rare [than the heavens] so obviously move with a more rapid motion than any earthy object); [let us concede that] the densest and heaviest objects have a proper motion of the quick and uniform kind which they suppose (although, again, as all agree, earthy objects are sometimes not readily moved even by an external force). Nevertheless, they would have to admit that the revolving motion of the earth must be the most violent of all motions associated with it, seeing that it makes one revolution in such a short time; the result would be that all objects not actually standing on the earth would appear to have the same motion, opposite to that of the earth: neither clouds nor other flying or thrown objects would ever be seen moving towards the east, since the earth's motion towards the east would always outrun and overtake them, so that all other objects would seem to move in the direction of the west and the rear. But if they said that the air is carried around in the same direction and with the same speed as the earth, the compound objects in the air would none the less always seem to be left behind by the motion of both [earth and air]; or if those objects too were carried around, fused, as it were, to the air, then they would never appear to have any motion either in advance or rearwards: they would always appear still, neither wandering about nor changing position, whether they were flying or thrown objects. Yet we quite plainly see that they do undergo all these kinds of motion, in such a way that they are not even slowed down or speeded up at all by any motion of the earth.

Ptolemy is probably summarizing the winning arguments in an old debate, going back as far as Aristarchus in about 240 B.C.:

ARCHIMEDES, *Psammites* (*Sand-reckoner*), c. 1, 1–10.

But Aristarchus of Samos brought out a book consisting of certain hypotheses, in which the premisses lead to the conclusion that the universe is many times greater than that now so called. His hypotheses are that the fixed stars and the sun remain motionless, that the earth revolves about the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface.

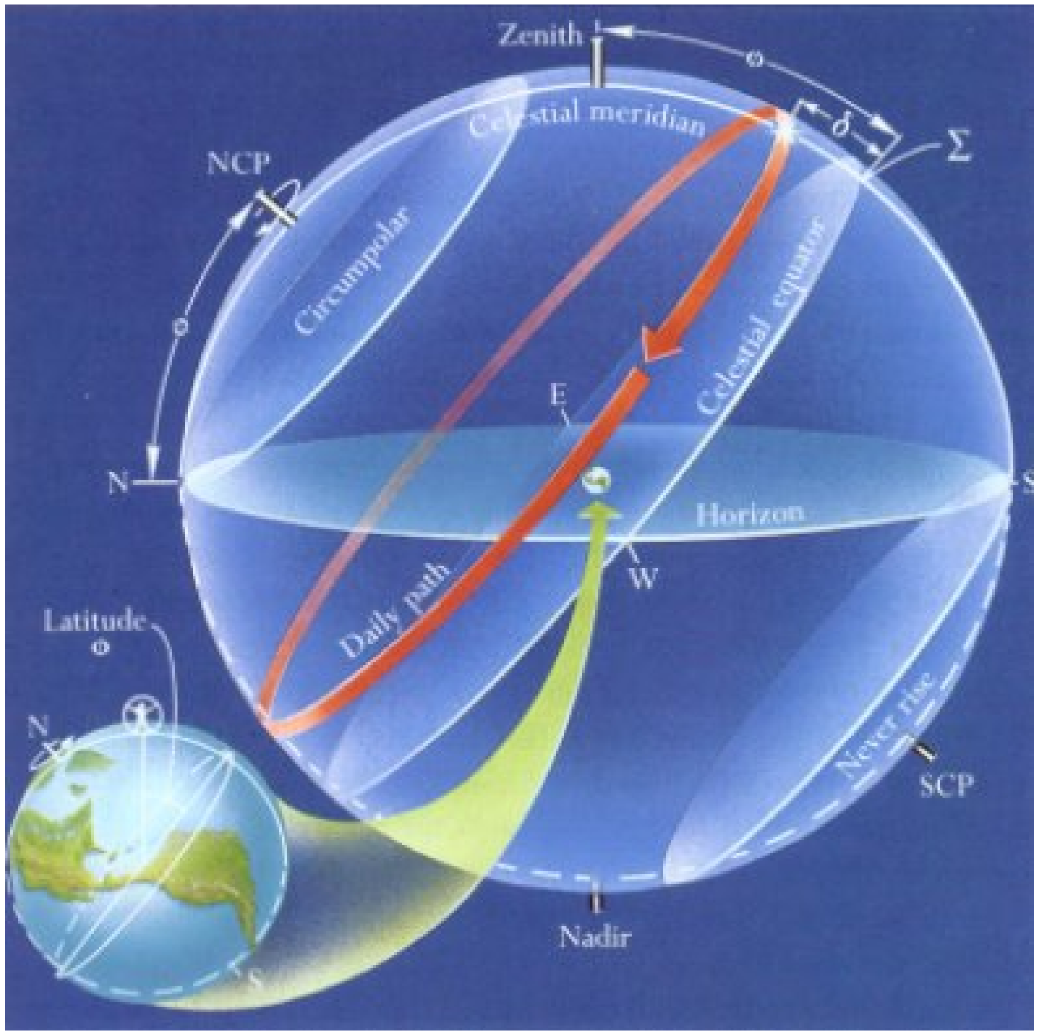
PLUTARCH

On the face in the moon

De facie in orbe lunae, cc. 5–10, 16, 21–22.

6. While I was still speaking, Pharnaces broke in: "Here, again, we have employed against us the stock device borrowed from the Academy, that of taking care, every time that they discuss things with others, not to allow their own opinions to be criticized, but always to put the others, whenever they meet them, in the position of defendants, not accusers. But you will not to-day draw me into defending the views you impute to the Stoics before you have rendered an account of your own action in turning the universe upside down." Lucius smiled and said: "Very well; only do not bring against me a charge of impiety such as Cleanthes used to say that it behoved Greeks to bring against Aristarchus of Samos for moving the Hearth of the Universe, because he tried to save the phenomena by the assumption that the heaven is at rest, but that the earth revolves in an oblique orbit, while also rotating about its own axis. Now we

Celestial Sphere

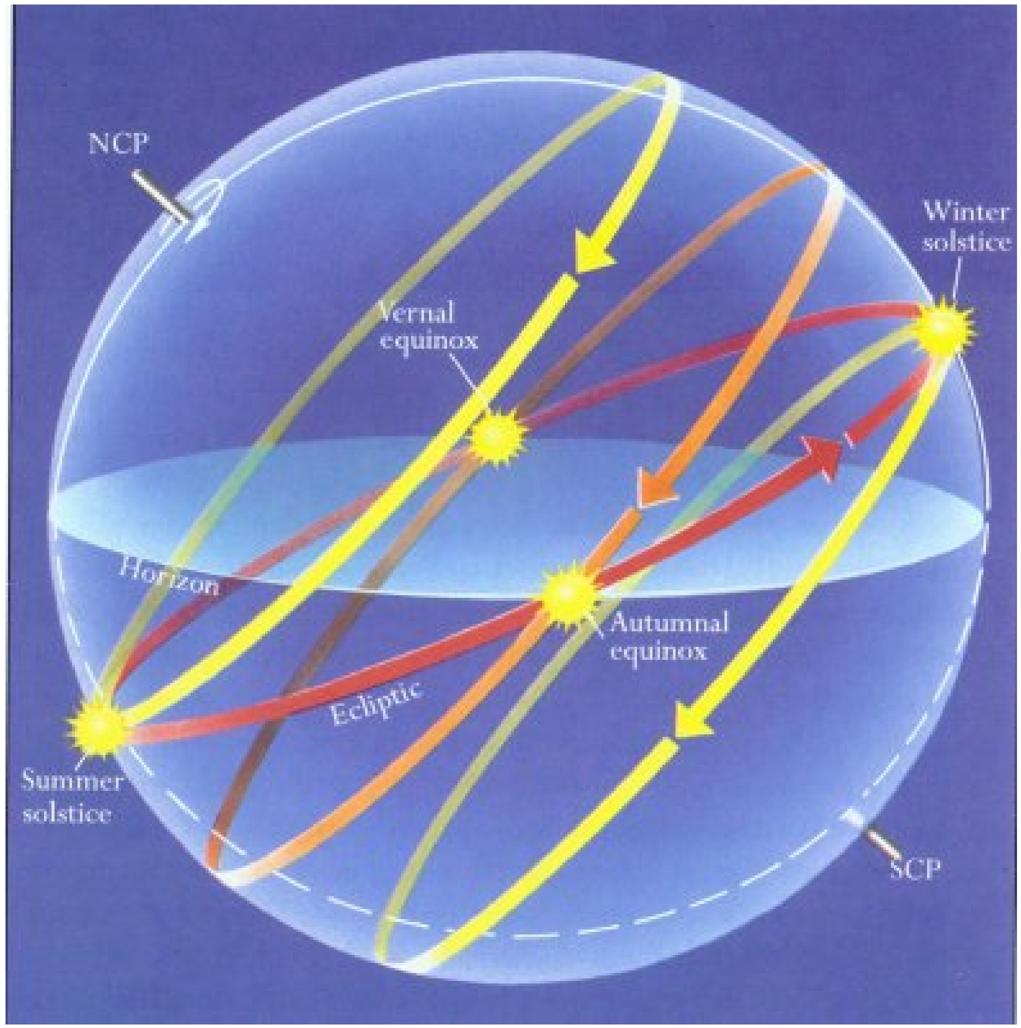


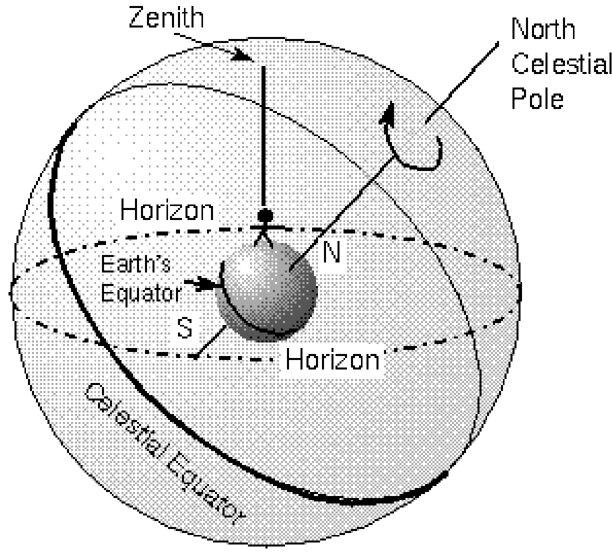
8. {*That there are two different primary motions in the heavens*}[43]

It was necessary to treat the above hypotheses first as an introduction to the discussion of particular topics and what follows after. The above summary outline of them will suffice, since they will be completely confirmed and further proven by the agreement with the phenomena of the theories which we shall demonstrate in the following sections. In addition to these hypotheses, it is proper, as a further preliminary, to introduce the following general notion, that there are two different primary motions in the heavens. One of them is that which carries everything from east to west: it rotates them with an unchanging and uniform motion along circles parallel to each other, described, as is obvious, about the poles of this sphere which rotates everything uniformly. The greatest of these circles is called the 'equator',[44] because it is the only [such parallel circle] which is always bisected by the horizon (which is a great circle), and because the revolution which the sun makes when located on it produces equinox everywhere, to the senses. The other motion is that by which the spheres of the stars perform movements in the opposite sense to the first motion, about another pair of poles, which are different from those of the first rotation.

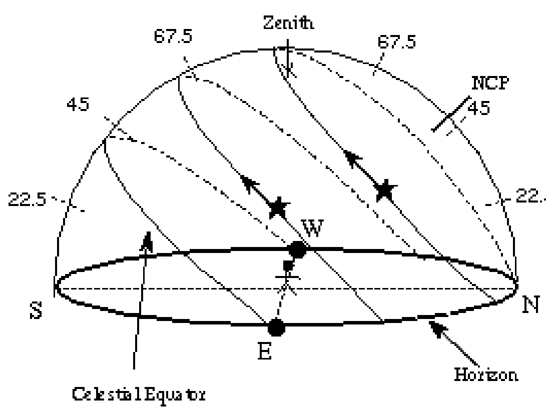
The second, multiple-part motion is encompassed by the first and encompasses the spheres of all the planets. As we said, it is carried around by the aforementioned [first motion], but itself goes in the opposite direction about the poles of the ecliptic, which are also fixed on the circle which produces the first motion, namely the circle through both poles [of ecliptic and equator]. Naturally they [the poles of the ecliptic] are carried around with it [the circle through both poles], and, throughout the period of the second motion in the opposite direction, they always keep the great circle of the ecliptic, which is described by that [second] motion, in the same position with respect to the equator.[48]

The oblique circle (the ecliptic, the path of the Sun, Moon and planets)

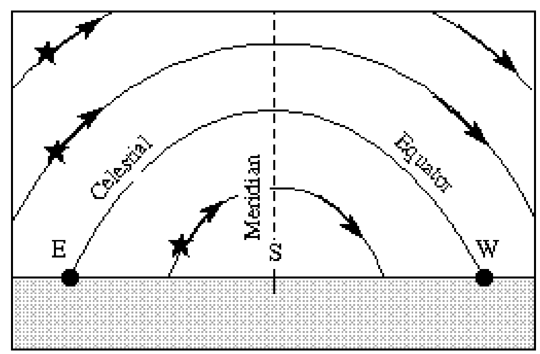




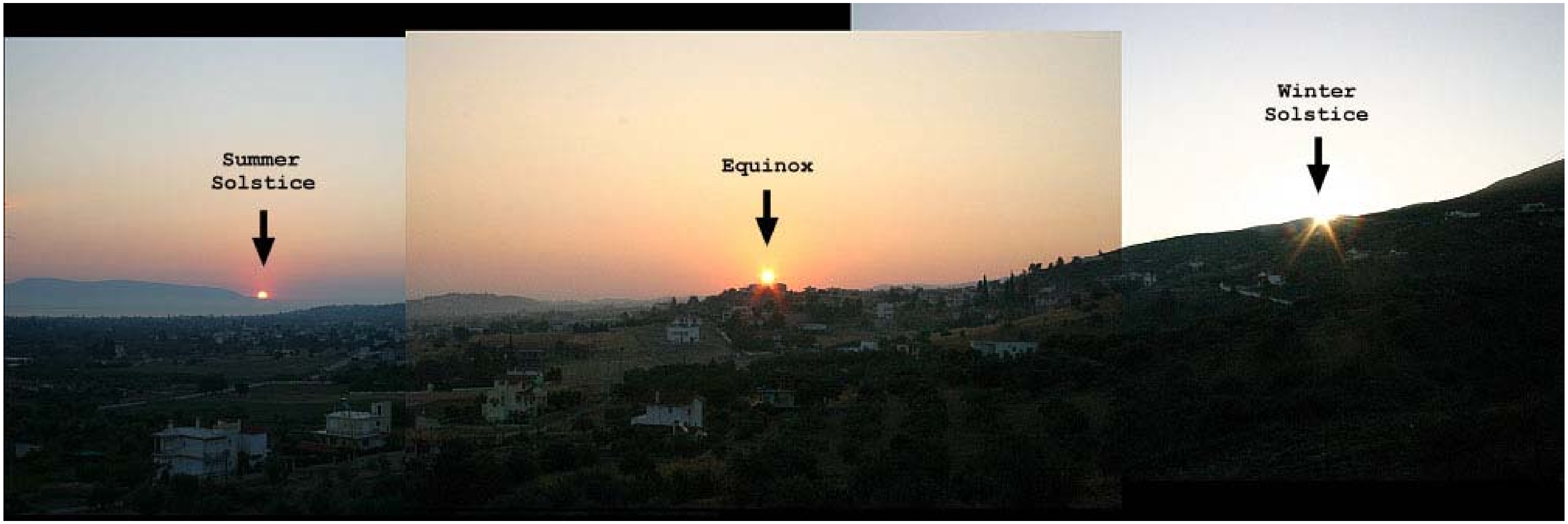
The celestial sphere for an observer in Seattle.
The angle between the zenith and the NCP = the angle between the celestial equator and the horizon.
That angle = 90° – observer's latitude.

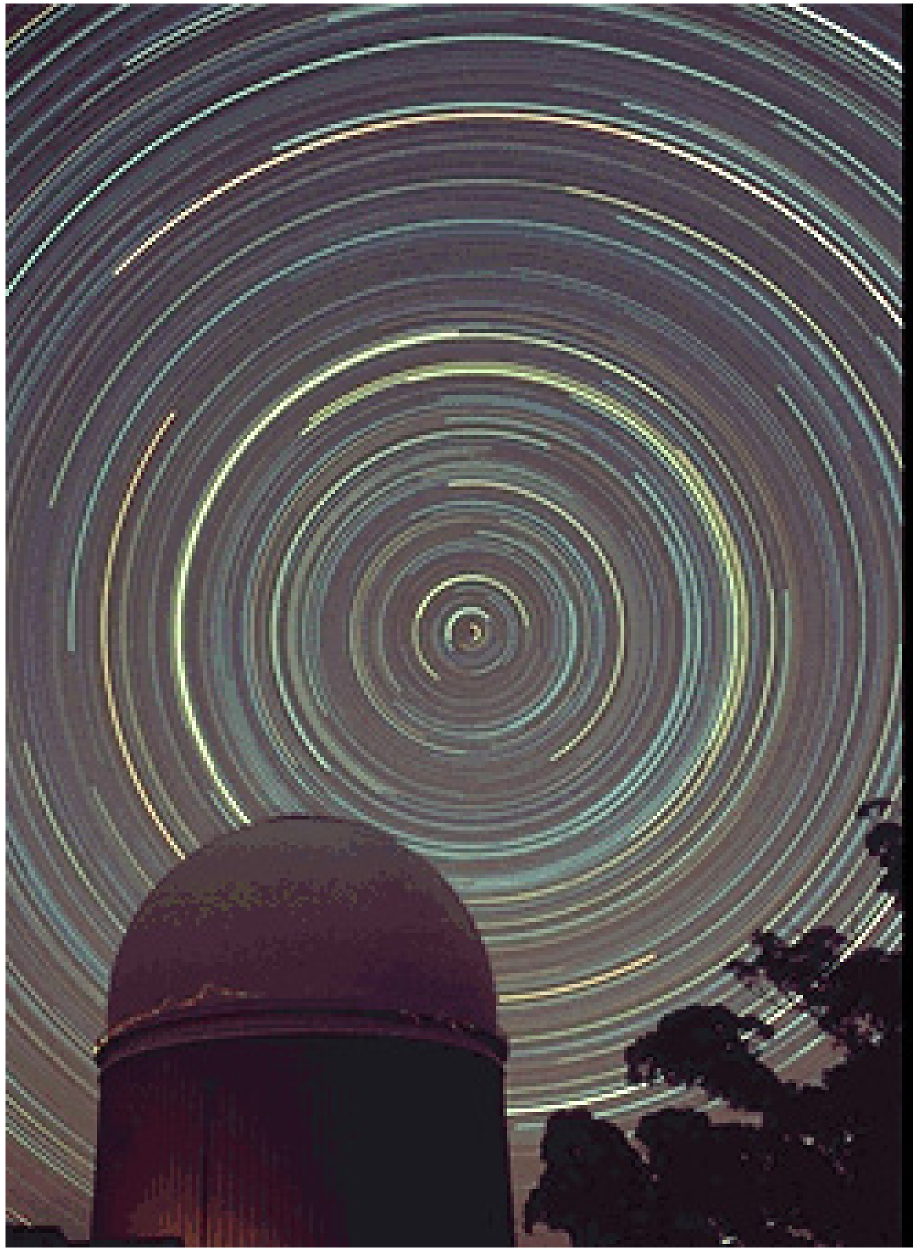


Stars motion at Seattle. Stars rotate parallel to the Celestial Equator, so they move at an angle with respect to the horizon here. Altitudes of 1/4, 1/2, and 3/4 the way up to the zenith are marked.

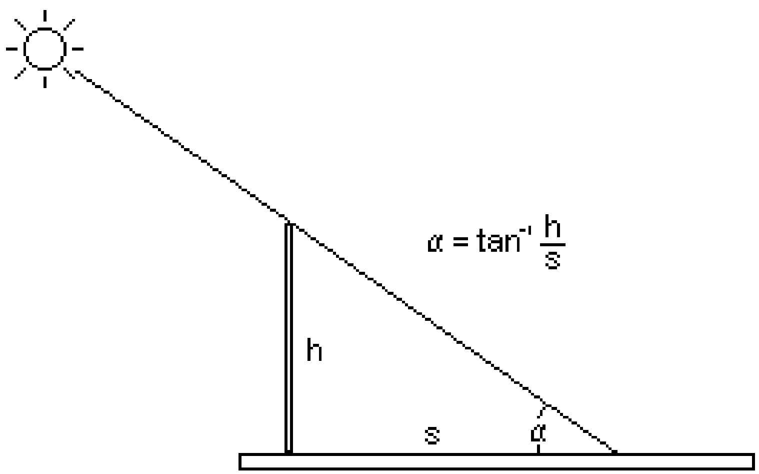


Your view from Seattle. Stars rise in the East half of the sky, reach maximum altitude when crossing the meridian (due South) and set in the West half of the sky. The Celestial Equator goes through due East and due West.

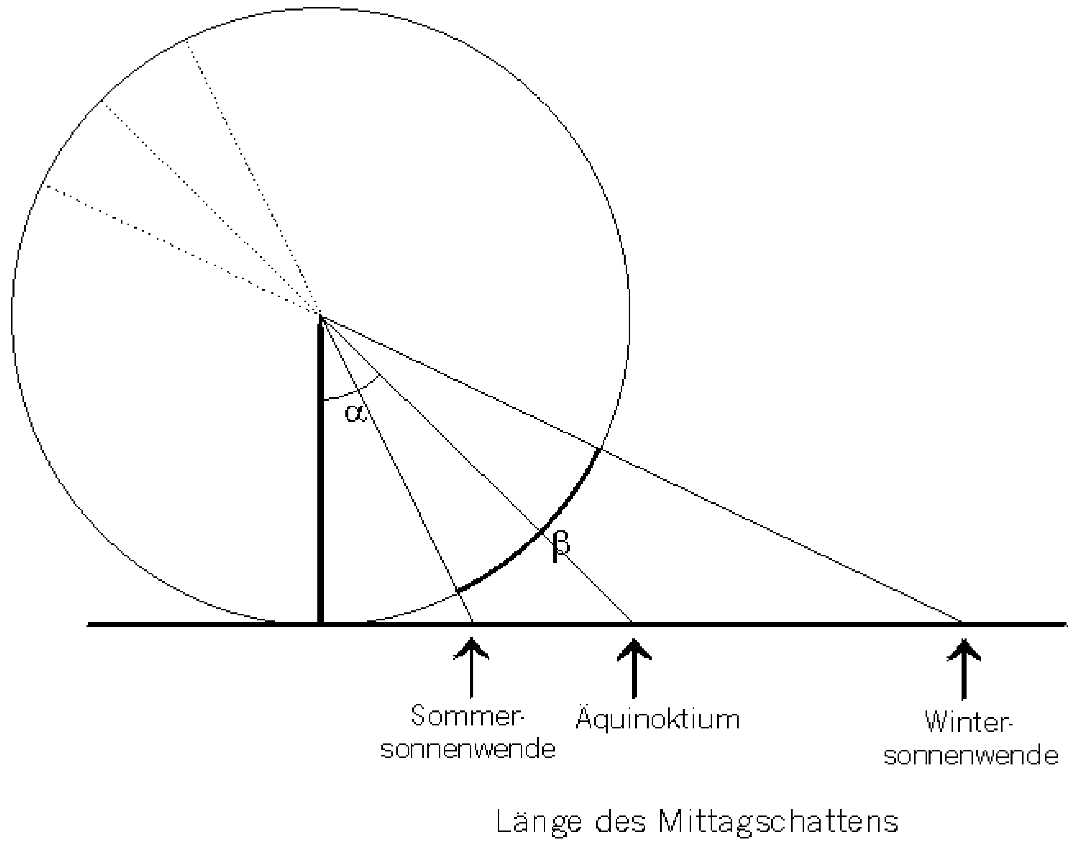

Summer
Solstice
Equinox
Winter
Solstice



Using a gnomon



Altitude by the Gnomon



α = geographical latitude

β = twice the obliquity of the ecliptic

Gnomon’s are also the basis of sundials:

How were these angles measured other than using a gnomon? Ptolemy describes two instruments:

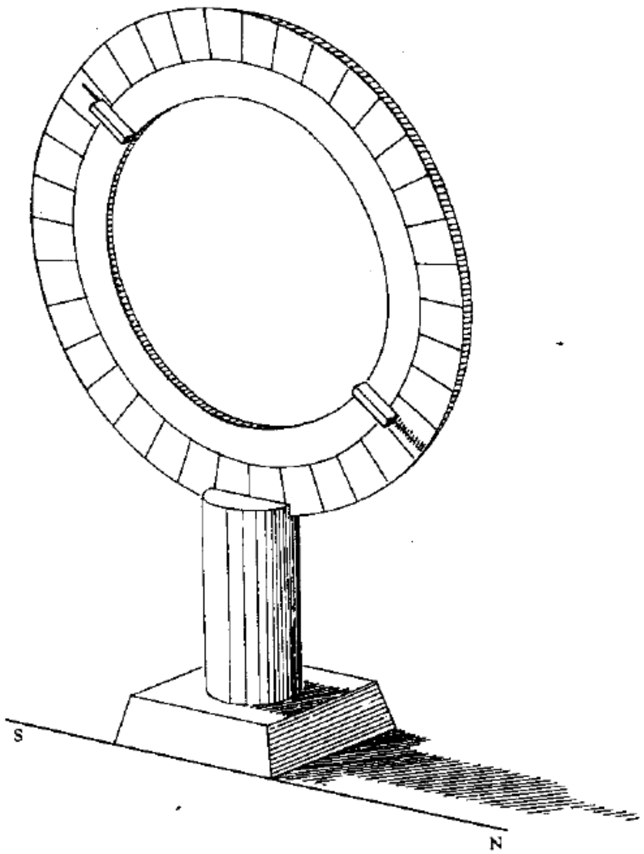


Fig. C

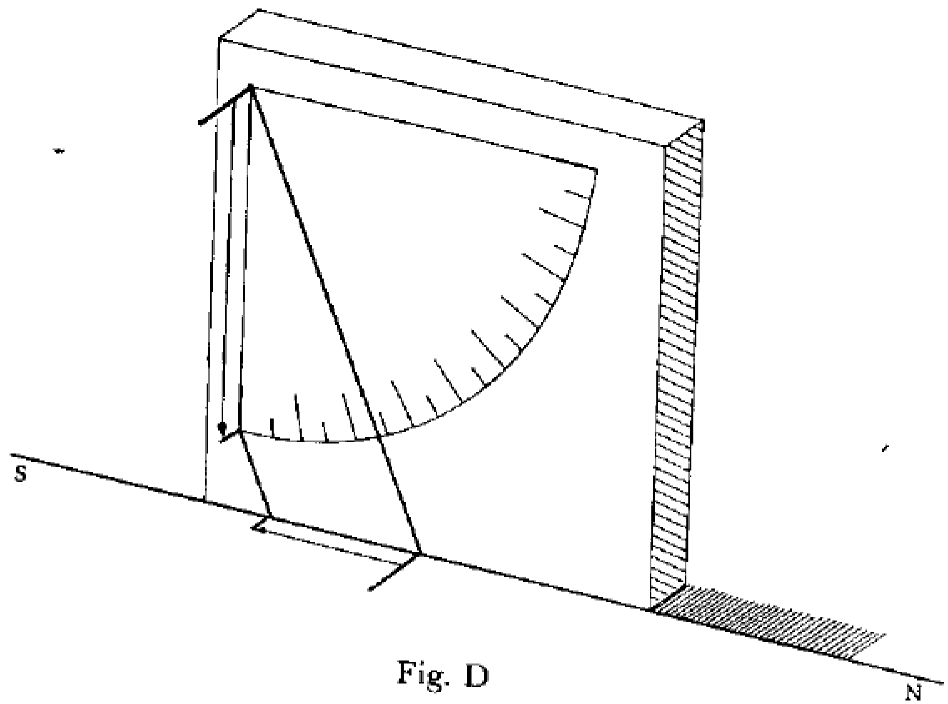


Fig. D

Expressing Numbers

Even today we measure angles in degrees, minutes, seconds, and we also measure time in hours, minutes, seconds.

In both cases there are 60 minutes per degree or hour, and 60 seconds per minute.

Apparently this began in Babylon, no later than early first millenium B.C. and probably a lot earlier, since we have many 1000's of surviving clay tablets covered with such numbers.

Ptolemy also used this base-60 sexagesimal number format, at least for the fractional part of the number. Thus he expressed the number $365+\frac{1}{4}-\frac{1}{300}$ as

$$\begin{aligned} 365+\frac{15}{60}-\frac{12}{3600} &= 365+\frac{14}{60}+\left(\frac{60}{3600}-\frac{12}{3600}\right) \\ &= 365+\frac{14}{60}+\frac{48}{3600} \\ &= 365;14,48 \end{aligned}$$

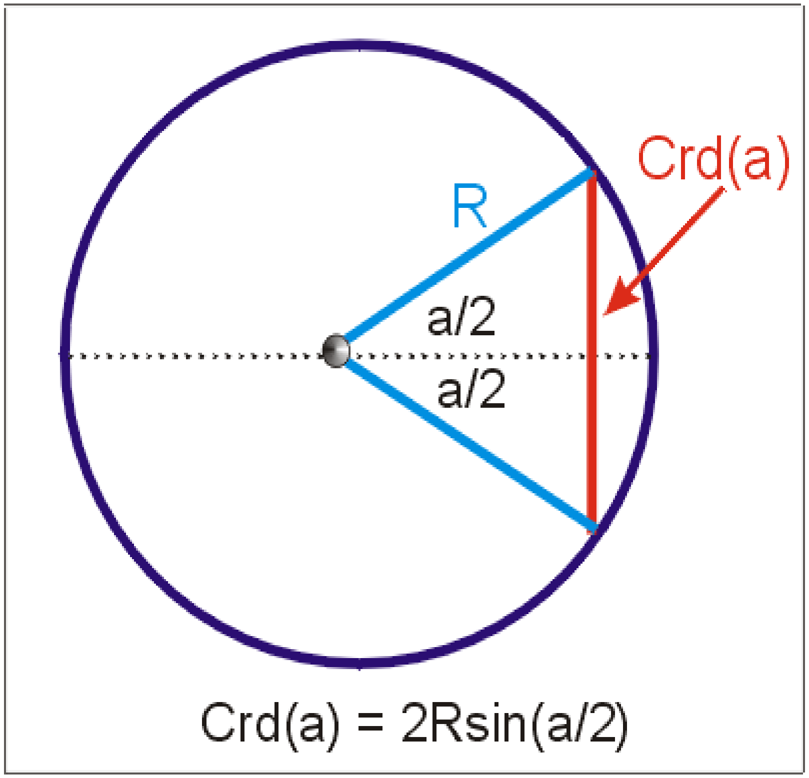
The integer part of the number was given in decimal.

1 = α	10 = ι	100 = ρ
2 = β	20 = κ	200 = σ
3 = γ	30 = λ	300 = τ
4 = δ	40 = μ	400 = υ
5 = ε	50 = ν	500 = φ
6 = ς (ϝ)	60 = ξ	600 = χ
7 = ζ	70 = ο	700 = ψ
8 = η	80 = π	800 = ω
9 = θ	90 = ϙ	900 = ϡ

With a good set of multiplication and division tables, which everyone had, manual arithmetic was no harder for them than it is for us.

Ptolemy used mostly plane geometry and trigonometry, with a little spherical trig when he needed it, which was not often.

For plane trig he had only one construct – the chord – rather than our sine, cosine, tangent, etc, and this was enough.



He also had good tables of the chord function, and was quite capable of interpolation, just as we (used to) do it.

TABLE OF CHORDS

Arcs	Chords	Sixtieths	Arcs	Chords	Sixtieths
½	0 31 25	1 2 50	23	23 55 27	1 1 33
1	1 2 50	1 2 50	23½	24 26 13	1 1 30
1½	1 34 15	1 2 50	24	24 56 58	1 1 26
2	2 5 40	1 2 50	24½	25 27 41	1 1 22
2½	2 37 4	1 2 48	25	25 58 22	1 1 19
3	3 8 28	1 2 48	25½	26 29 1	1 1 15
3½	3 39 52	1 2 48	26	26 59 38	1 1 11
4	4 11 16	1 2 47	26½	27 30 14	1 1 8
4½	4 42 40	1 2 47	27	28 0 48	1 1 4
5	5 14 4	1 2 46	27½	28 31 20	1 1 0
5½	5 45 27	1 2 45	28	29 1 50	1 0 56
6	6 16 49	1 2 44	28½	29 32 18	1 0 52
6½	6 48 11	1 2 43	29	30 2 44	1 0 48
7	7 19 33	1 2 42	29½	30 33 8	1 0 44
7½	7 50 54	1 2 41	30	31 3 30	1 0 40
8	8 22 15	1 2 40	30½	31 33 50	1 0 35
8½	8 53 35	1 2 39	31	32 4 8	1 0 31
9	9 24 54	1 2 38	31½	32 34 22	1 0 27

10. {*On the size of chords*}[50]

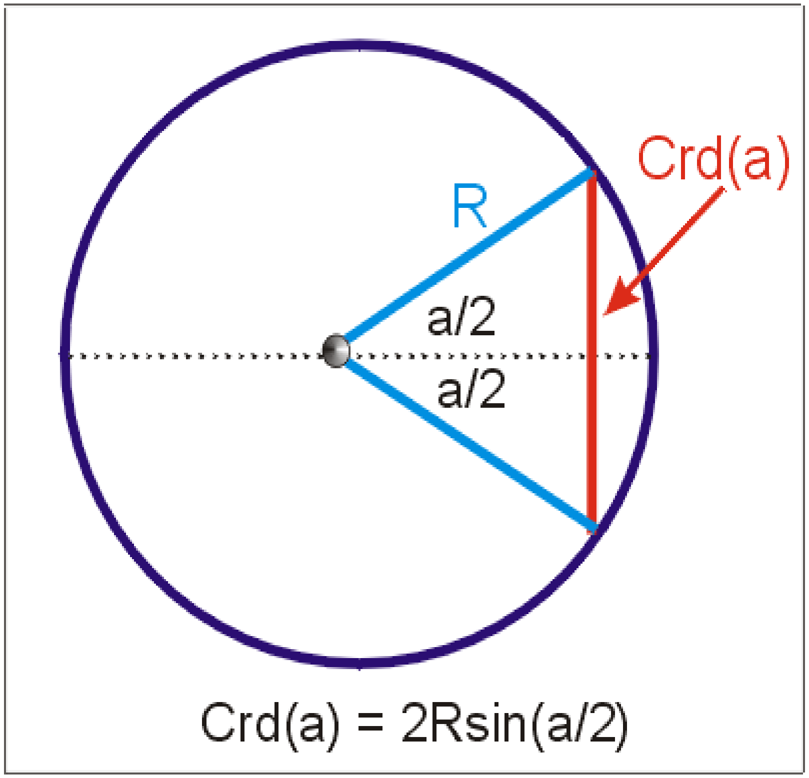
For the user's convenience, then, we shall subsequently set out a table of their amounts, dividing the circumference into 360 parts, and tabulating the chords subtended by the arcs at intervals of half a degree, expressing each as a number of parts in a system where the diameter is divided into 120 parts. [We adopt this norm] because of its arithmetical convenience,[51] which will become apparent from the actual calculations. But first we shall show how one can undertake the calculation of their amounts by a simple and rapid method, using as few theorems as possible, the same set for all. We do this so that we may not merely have the amounts of the chords tabulated unchecked, but may also readily undertake to verify them by computing them by a strict geometrical method. In general we shall use the sexagesimal system for our arithmetical computations, because of the awkwardness of the [conventional] fractional system. Since we always aim at a good approximation, we will carry out multiplications and divisions only as far as to achieve a result which differs from the precision achievable by the senses by a negligible amount.

Ptolemy says that he will present a 'simple and efficient' way to compute the chords, but he doesn't actually say the table *was* computed that way, or even that he computed it. In fact, there is good reason to think that it was *not* computed using his methods, or that he was the person who computed it. Unfortunately, however, we have no evidence about who did compute it.

As we will see in Lectures 2 and 3, it is likely that Hipparchus also had a good command of trigonometry, both plane and spherical, but he also probably had a simpler trig table. Most people assume he also used the chord construct, but there is no evidence for this, and there is some reason to think he used instead the sine.

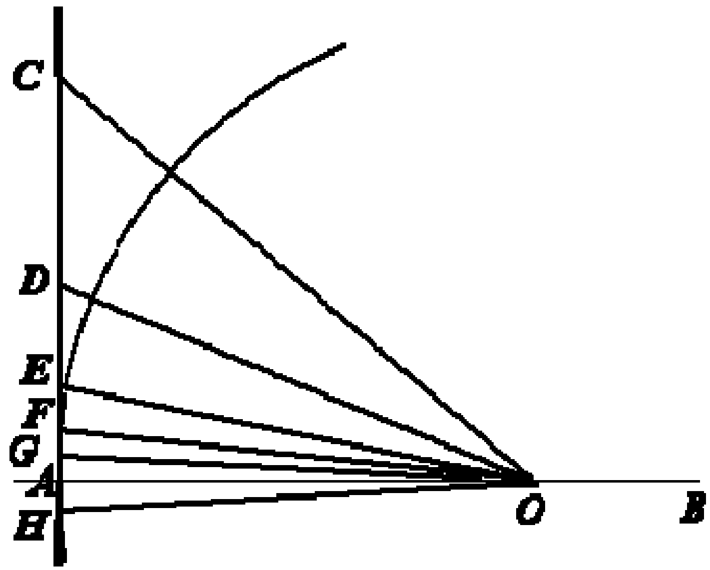
$$R = \frac{360^\circ \cdot 60'/^\circ}{2\pi} = \frac{21,600}{2\pi} \simeq 3438$$

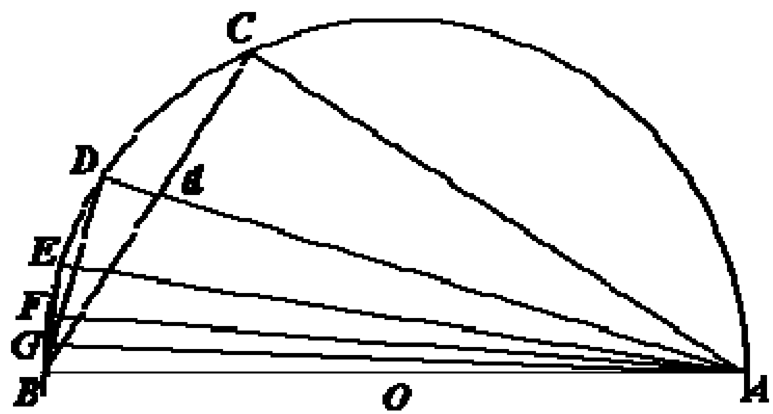
$$D = \frac{21600}{\pi} \simeq 6875$$



Angle(degrees)	Chord
0	0
7 ½	450
15	897
22 ½	1341
30	1780
37 ½	2210
45	2631
52 ½	3041
60	3438
67 ½	3820
75	4186
82 ½	4533
90	4862
97 ½	5169
105	5455
112 ½	5717
120	5954
127 ½	6166
135	6352
142 ½	6511
150	6641
157 ½	6743
165	6817
172 ½	6861
180	6875

There is also no reason to think that Hipparchus invented trignonometry and tables, either chord or sine. In fact, a work of Archimedes shows the explicit computation of about 2/3's of the entries in Hipparchus' (supposed) table, and computing the other entries would be straightforward.





Archimedes gets $\frac{66}{2017\frac{1}{4}} < \sin 1\tfrac{7}{8}^\circ < \frac{153}{4673\frac{1}{2}}$ (equivalent to $0.03272 < \sin 1\tfrac{7}{8}^\circ < 0.03274$)

which leads to

$$3\frac{10}{71} < \pi < 3\frac{1}{7}$$

	circumscribed		inscribed		circumscribed	inscribed
Angle	*a*	*c*	*a*	*c*	Base 3438	Base 3438
3 6/8	153	2339 3/8	780	11926	225	225
7 4/8	153	1172 1/8	780	5975 7/8	449	449
11 2/8	169	866 2/8	70	358 7/8	671	670
15	153	591 1/8	780	3013 6/8	890	890
18 6/8	571	1776 2/8	2911	9056 1/8	1105	1105
22 4/8	169	441 5/8	70	182 7/8	1315	1316
26 2/8	744	1682 3/8	3793 6/8	8577 3/8	1520	1520
30	153	306	780	1560	1719	1719
33 6/8	408	734 3/8	169	304 2/8	1910	1909
37 4/8	571	937 7/8	2911	4781 7/8	2093	2093
41 2/8	1162 1/8	1762 3/8	5924 6/8	8985 6/8	2267	2267
45	169	239	70	99	2431	2431

- columns 2–5 come from Archimedes, while columns 6–7 are just

$$\frac{a}{c}\times 3{,}438$$

- notice that Archimedes is working entirely in sine and cosine, never chord
- there is no doubt that Hipparchus was familiar with Archimedes' work on this
- about all we can conclude is that Archimedes, Hipparchus, or someone in between *might* have computed the first trig table this way

We can, in fact, go even farther back into the very early history of trigonometry by considering Aristarchus' *On Sizes and Distances*, and we shall see that a plausible case can be made that his paper could easily have been the inspiration for Archimedes' paper. The problem Aristarchus posed was to find the ratio of the distance of the Earth to the Moon to the distance of the Earth to the Sun [as we will see in Lecture 4]. He solved this problem by assuming that when that the Moon is at quadrature, meaning it appears half-illuminated from Earth and so the angle Sun-Moon-Earth is 90°, the Sun-Moon elongation is 87°, and so the Earth-Moon elongation as seen from the Sun would be 3°. Thus his problem is solved if he can estimate the ratio of opposite side to hypotenuse for a right triangle with an angle of 3°, or simply what we call sin 3°.

Aristarchus proceeded to solve this problem is a way that is very similar to, but not as systematic as, the method used by Archimedes. By considering circumscribed (Fig. 2 below) and inscribed triangles (Fig 3 below) and assuming a bound on $\sqrt{2}$ Aristarchus effectively establishes bounds on sin 3° as

$$\frac{1}{20} < \sin 3^\circ < \frac{1}{18}$$

and, although he does not mention it, this also establishes bounds on π as

$$3 < \pi < 3\tfrac{1}{3}$$

clearly not as good as Aristarchus got just a few years later.

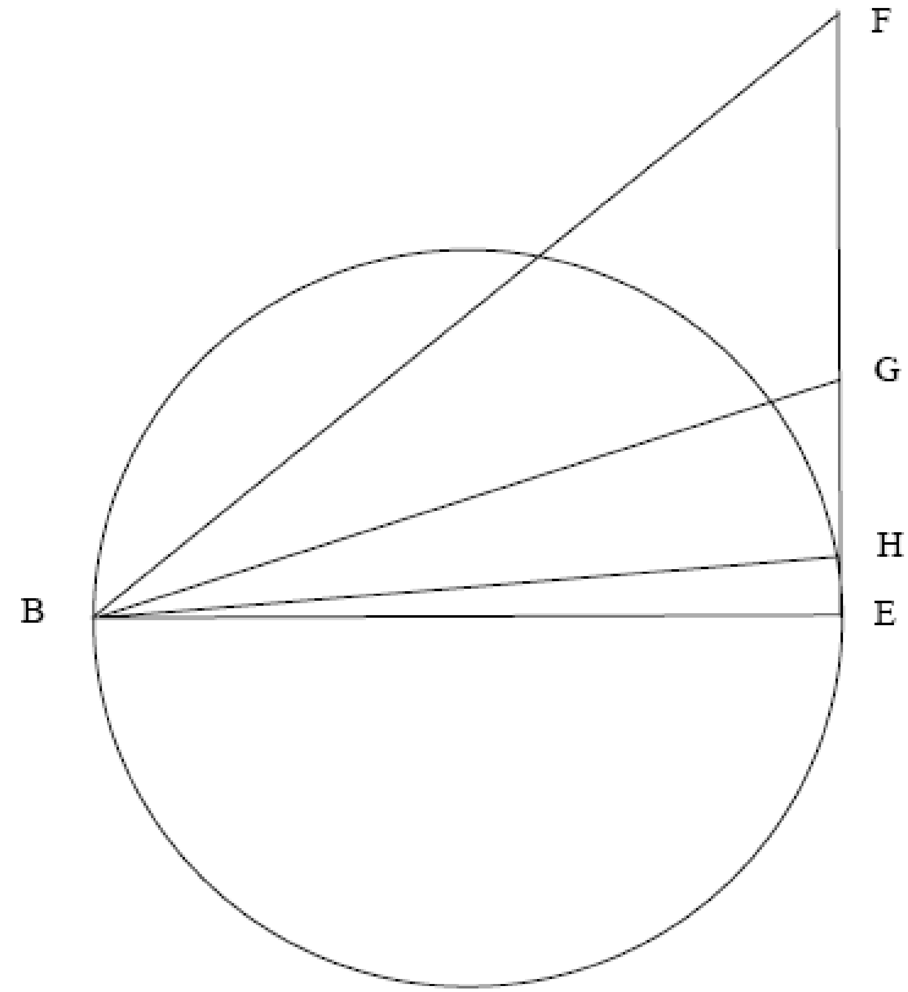


Figure 2. BE is a diameter of the circle, angle EBF is 45°, angle EBG is 22½°, and angle EBH is 3° (not to scale). Since EBG/EBH = 15/2 then GE/EH > 15/2. Since FG/GE = $\sqrt{2}$ > 7/5 then FE/EG > 12/5 = 36/15 and so FE/EH > (36/15)(15/2) = 18/1.

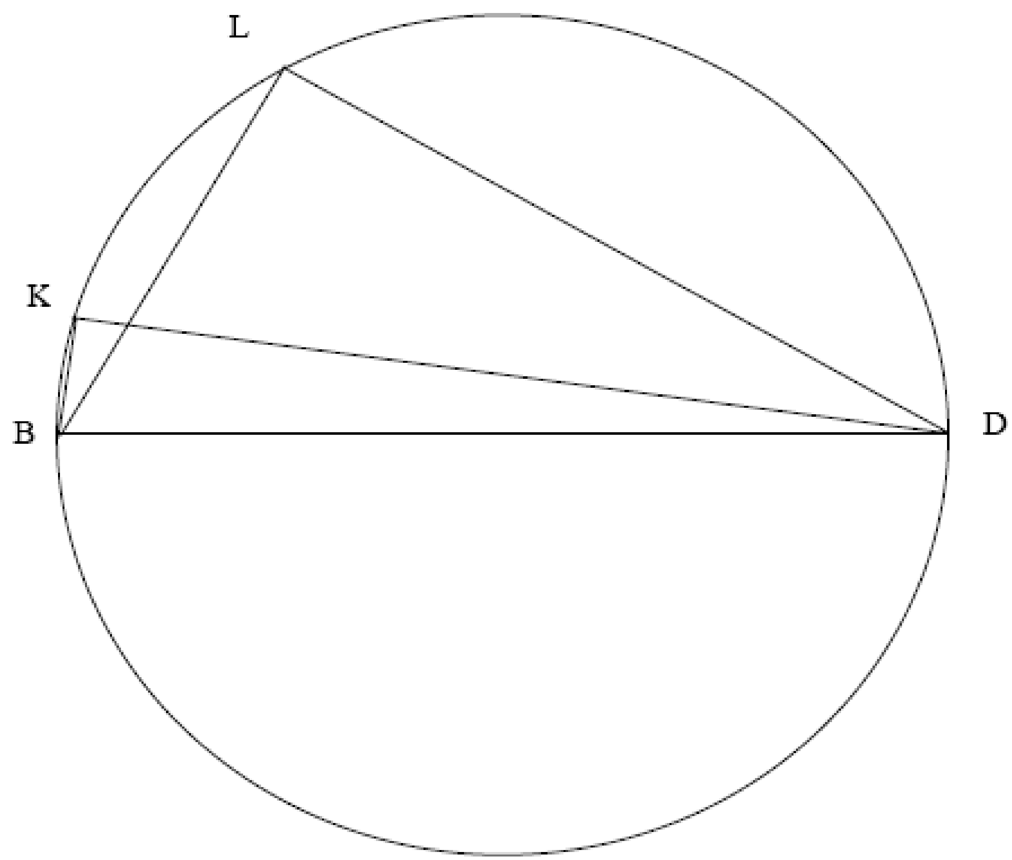


Figure 3. BD is a diameter of the circle, angle BDL = 30°, and angle BDK = 3° (not to scale). Since arc BL = 60° and arc BK = 6° then BL/BK < 10/1. Since BD = 2 BL then BD/BK < 20/1.

Actually, the sine (not chord!) table that we suppose was used by Hipparchus shows up clearly in Indian astronomical texts of the 5th and 6th centuries A.D. For example, Aryabhata writes in *The Aryabhatiya* (ca. A.D. 500) verse I.10:

> *10. The sines reckoned in minutes of arc are 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22,7.*

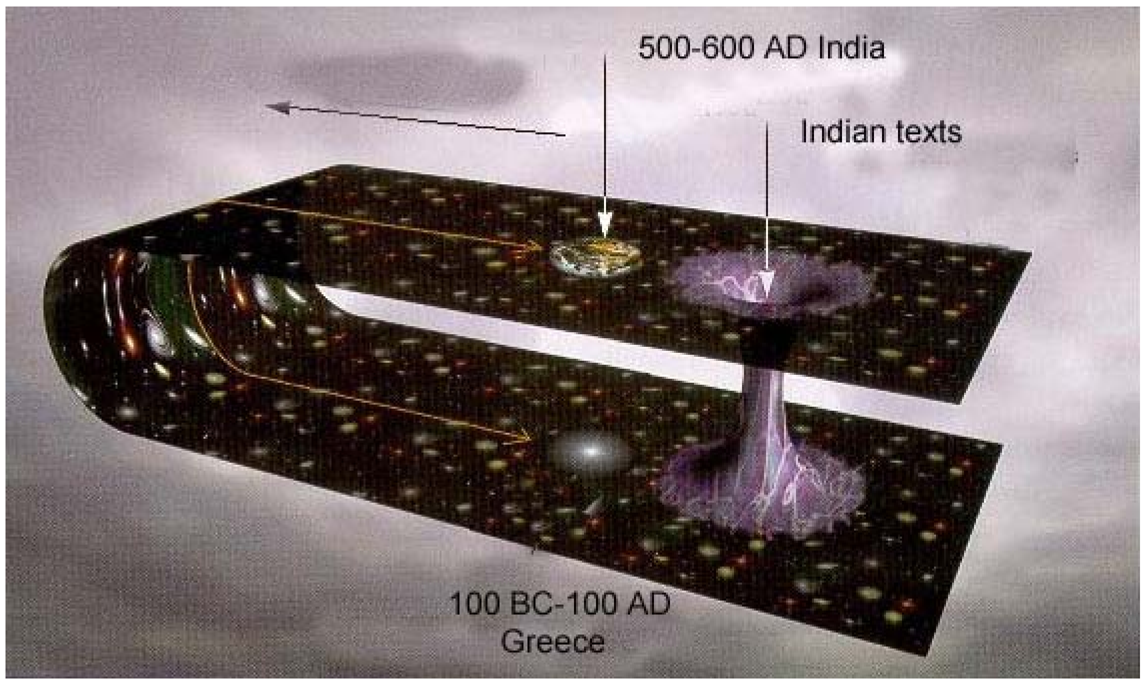
and later he explains how to compute these in verse II.12:

> *12. By what number the second sine is less than the first sine, and by the quotient obtained by dividing the sum of the preceding sines by the first sine, by the sum of these quantities the following sines are less than the first sine.*

These are clearly not sines but rather the *differences* of adjacent terms in the table of sines. The base is 3,438, just as Hipparchus used.

Many similar examples (to be seen in coming weeks) lead to what I call the Neugebauer – Pingree – van der Waerden Hypothesis:

> *The texts of ancient Indian astronomy give us a sort of wormhole through space-time back into an otherwise inaccessible era of Greco-Roman developments in astronomy.*



Thus the essentially universally accepted view that **the astronomy we find in the Indian texts is pre-Ptolemaic**. Summarizing the prevailing opinion, Neugebauer wrote in 1956:

> "Ptolemy's modification of the lunar theory is of importance for the problem of transmission of Greek astronomy to India. The essentially Greek origin of the Surya-Siddhanta and related works cannot be doubted – terminology, use of units and computational methods, epicyclic models as well as local tradition – all indicate Greek origin. But it was realized at an early date in the investigation of Hindu astronomy that the Indian theories show no influence of the Ptolemaic refinements of the lunar theory. **This is confirmed by the planetary theory, which also lacks a characteristic Ptolemaic construction, namely, the "*punctum aequans*,"** to use a medieval terminology".

This fundamental idea will be explored much further in coming lectures.

Ptolemy's obliquity and latitude of Alexandria

From observations of this kind, and especially from comparing observations near the actual solstices, which revealed that, over a number of returns [of the sun], the distance from the zenith was in general the same number of degrees of the meridian circle at the [same] solstice, whether summer or winter, we found that the arc between the northernmost and southernmost points, which is the arc between the solstitial points, is always greater than $47\frac{2}{3}°$ and less than $47\frac{3}{4}°$. From this we derive very much the same ratio as Eratosthenes, which Hipparchus also used. For [according to this] the arc between the solstices is approximately 11 parts where the meridian is 83.[75]

Ptolemy uses $2\varepsilon = 47°;42',30''$ but in reality he should have gotten about 47°;21′. Now 21′ is a fairly large error for this kind of measurement, about 2/3rd the size of the Moon. What is not surprising is that Ptolemy made such an error, but that he got *exactly* the same values used by Eratosthenes and Hipparchus, who should have gotten about 47°;27′.

This kind of thing occurs frequently throughout the *Almagest*.

For the geographical latitude, Ptolemy writes:

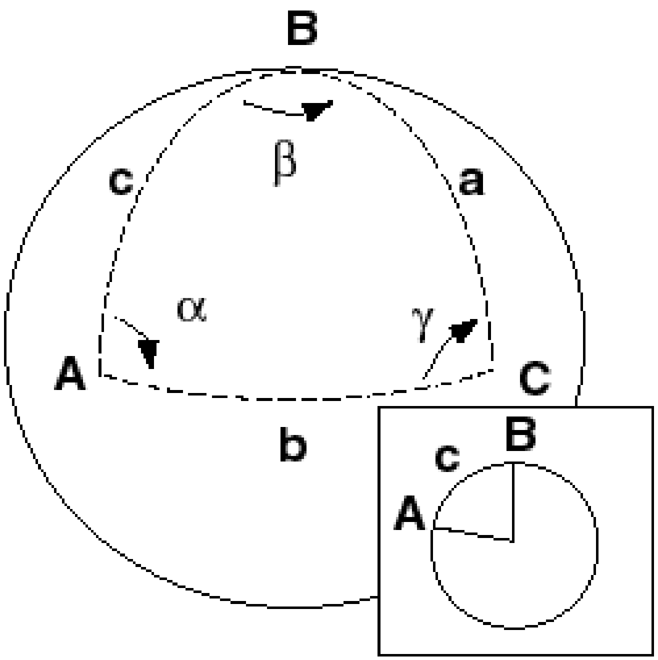
> From the preceding kind of observation it is easy to derive immediately the latitude of the region in which the observation is made, wherever it is: one takes the point halfway between the two extrema; this point lies on the equator; then one takes the distance between this point and the zenith, which is the same, obviously, as the distance of the poles from the horizon.

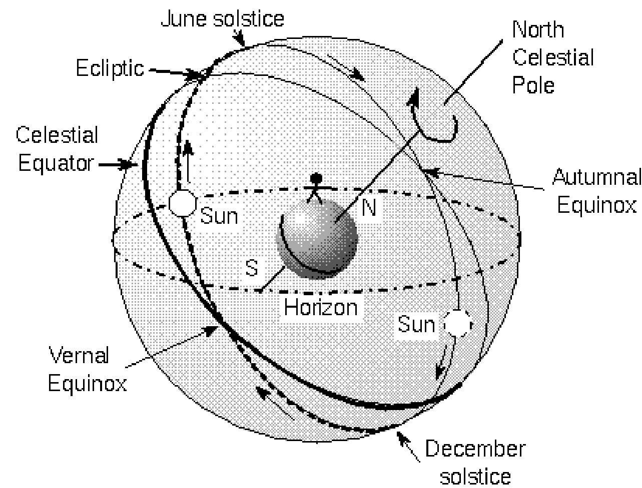
and later in *Almagest* 5.12:

> latitude either side of the ecliptic is shown to be 5°. For the zenith distance of the equator at Alexandria has been shown to be 30;58°; if we subtract from this the

Actually, the latitude of Alexandria is between 31°;13′and 31°;19′,depending on exactly where Ptolemy worked (probably closer to the more northern limit). Ptolemy's value 30°;58′follows exactly from an equinoctial shadow ratio of 5/3, and was probably also a value he inherited from some old tradition.

Spherical trigonometry solves problems related to circles on a sphere.





The Sun moves among the stars along the ecliptic, completing one 360° path in one year. The ecliptic is tilted by 23.5° with respect to the celestial equator. The Sun's position on the celestial sphere in April (full circle) and in October (dashed circle) is shown.

A particular problem is to compute the angles between the ecliptic, the equator, and the horizon. Another is to compute the time required for a given segment of the ecliptic to rise or set above or below the horizon. Another is to compute the length of the longest (or shortest) day at any given geographic latitude.

from *Almagest* Book 2.6, for the parallel of the Tropic of Cancer:

7. The seventh is the parallel with a longest day of $13\frac{1}{2}$ equinoctial hours. This is 23;51° from the equator[34] and goes through Soene.[35] This is the first of the so-called 'one-way-shadow'[36] parallels. For in this region the noon shadows of the gnomon never point towards the south. Only at the actual summer solstice does the sun come into the zenith for those beneath this parallel, so that the gnomons appear shadowless. For they are exactly the same distance from the equator as the summer solstice is. At every other time the shadows of the gnomons point towards the north. In this region, for a gnomon of 60^p, the equinoctial shadow is $26\frac{1}{2}^p$, the winter [solsticial] shadow is $65\frac{5}{6}^p$, and the summer [solsticial] shadow is zero.[37] Furthermore, all parallels north of this up to the northern boundary of our part of the inhabited world have the shadows going one way. For in those regions the gnomons at noon neither become shadowless nor point their shadows towards the south: they always point them towards the north, since the sun never comes into the zenith for them, either.

and some parallels further north:

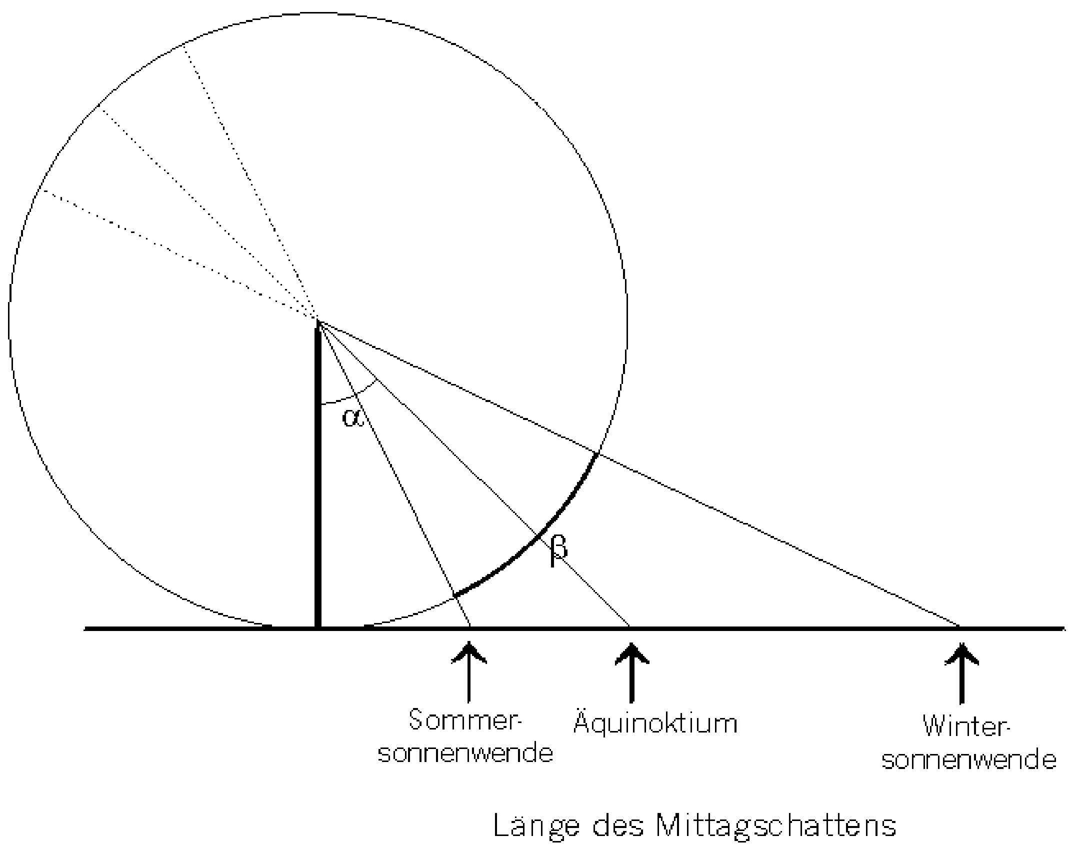
10. The tenth is the parallel with a longest of $14\frac{1}{4}$ equinoctial hours. This is 33;18° from the equator, and goes through the middle of Phoenicia. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is 10^p, the equinoctial shadow $39\frac{1}{2}^p$, and the winter [solsticial] shadow $93\frac{1}{12}^p$.[40]

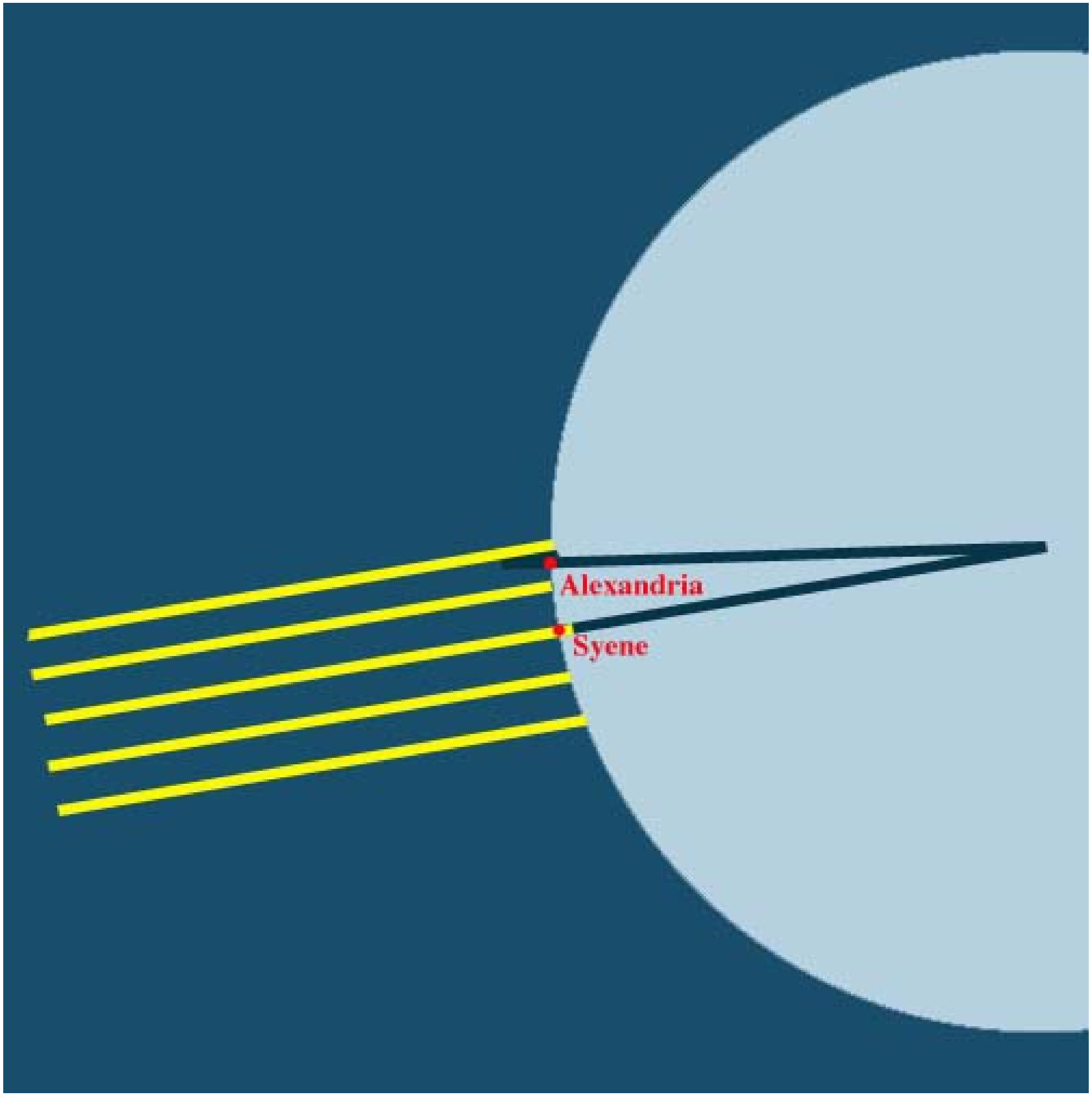
11. The eleventh is the parallel with a longest day of $14\frac{1}{2}$ equinoctial hours. This is 36° from the equator, and goes through Rhodes. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $12\frac{11}{12}^p$, the equinoctial shadow $43\frac{3}{5}^p$,[41] and the winter [solsticial] shadow $103\frac{1}{3}^p$.

12. The twelfth is the parallel with a longest day of $14\frac{3}{4}$ equinoctial hours. This is 38;35° from the equator, and goes through Smyrna. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $15\frac{2}{3}^p$, the equinoctial shadow is $47\frac{5}{6}^p$, and the winter [solsticial] shadow is $114\frac{11}{12}^p$.

13. The thirteenth is the parallel with a longest day of 15 equinoctial hours. This is 40;56° from the equator, and goes through the Hellespont. In this region, for a gnomon of 60^p, the summer [solsticial] shadow is $18\frac{1}{2}^p$, the equinoctial shadow $52\frac{1}{6}^p$, and the winter [solsticial] shadow $127\frac{5}{6}^p$.[42]

so Ptolemy is systematically *computing* what the shadow lengths will be at a sequence of geographical longitudes from the equator to the arctic circle.





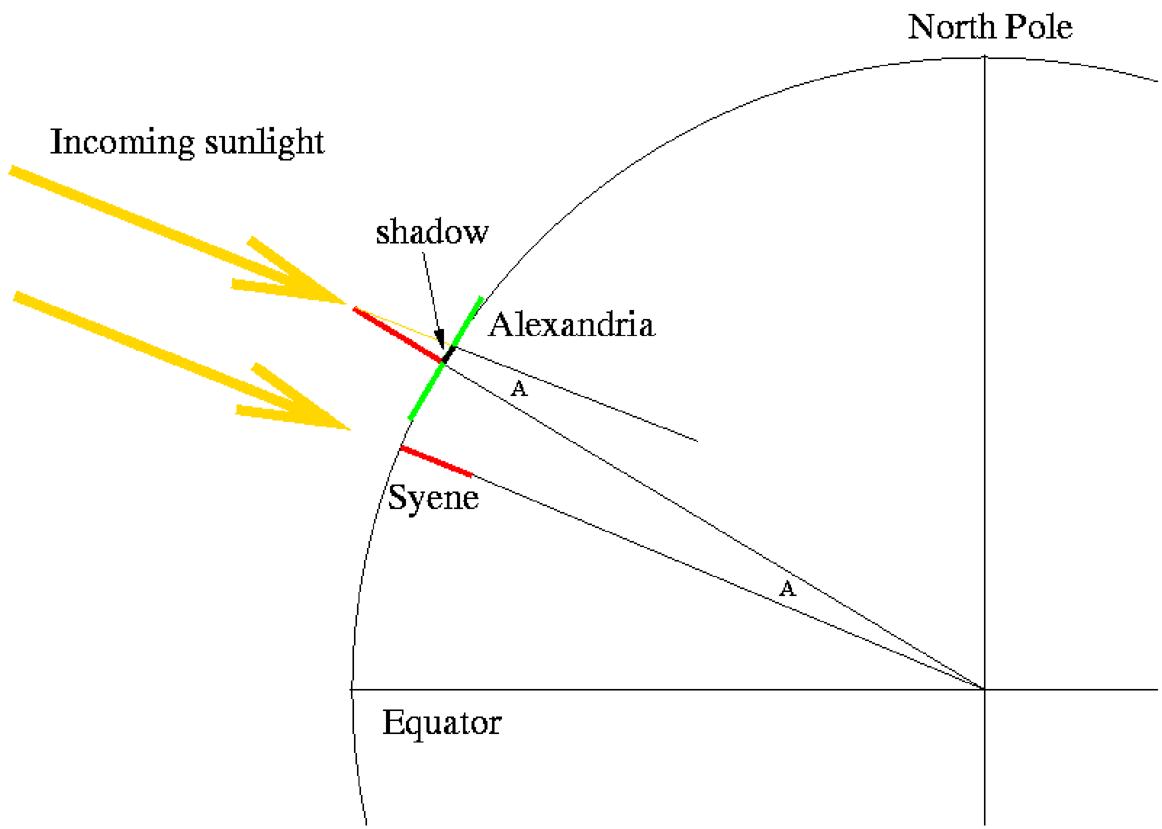
This had been going on for centuries. In about 200 B.C. Eratosthenes had managed to determine the circumference of the Earth.

Strabo, writing about A.D. 5, gives and interesting account of the work of both Eratosthenes and Hipparchus in this area (see the supplementary reading).

Eratosthenes is said to have measured the angle as 7 1/5 degrees, and took the distance from Syene to Alexandria as 5,000 stades, giving

$$C_{Earth} = \frac{360}{7\frac{1}{5}} \times 5{,}000 = 50 \times 5{,}000 = 250{,}000 \text{ stades}$$

which he rounded to 252,000 stades to make it divisible by 60 (and also 360).



Lecture 2

- *Almagest* Book 3
- the length of the year
- the length of the seasons
- the geometric models
- the length of the day
- the background
- lost episodes in solar history