ISC 5935 - Computational Tools for Finite Elements
Homework #10
Assigned 12 November 2014, Due 19 November 2014
http://people.sc.fsu.edu/∼jburkardt/classes/fem_2014/homework10.pdf

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1. Suppose that we are solving the following version of the time dependent heat equation in one spatial dimension:

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x} k(x) \frac{\partial u}{\partial x} = f(x,t)$$

over the space interval $[0,4]$ and time interval $[0,5]$, where $f(x,t) = 0$ and $k(x) = e^x$ and we will not worry about the details of the initial and boundary conditions.

If the time derivative is handled using an explicit Euler scheme, and we use a finite difference scheme with a spatial mesh of size Δx, what does the CFL condition specify as the upper limit on Δt?

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2. Now suppose that we solve the system in problem #1 with $\Delta x = 0.25$ and the maximum possible Δt allowable by the CFL, and suppose we are able to show that the L2 norm (over the time and space domain) is 0.016. Moreover, suppose we know that the error is proportional to $O(h^2) + O(t^2)$. If we simply want to reduce the error to 0.001, we need to reduce both Δx and Δt by a factor of 1/4. But if we reduce Δx, the new Δt must also be small enough to satisfy the new CFL condition.

If we reduce Δx by 1/4, by what factor must Δt be reduced, just to satisfy the CFL condition?

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3. Write a finite difference code that uses the explicit forward Euler time discretization for $\frac{\partial u}{\partial t} \approx \frac{u_{i,j+1} - u_{i,j}}{\Delta t}$ and the usual centered space discretization $\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{\Delta x^2}$ for the time dependent heat equation in 1D. Use $k(x) = 1$ and $f(x,t) = 0$, let the space interval be $0 < x < 1$, and the time interval be $0 < t < 0.10$.

The initial condition should be

$$u(x,0) = u0(x) = 0.75 e^{-(\frac{x-0.5}{0.1})^2}$$

and the boundary conditions for all time should be

$$u(0.0,t) = u0(0.0) = 0.75 e^{-(\frac{-0.5}{0.1})^2}$$
$$u(1.0,t) = u0(1.0) = e^{-(\frac{0.5}{0.1})^2}$$

a) Compute $u(x,t)$, using $\Delta x = \frac{1}{20}$ and $\Delta t = \frac{1}{1000}$. and make plots at $t = 0.00$, 0.05, and 0.10.

b) Repeat a), but now use $\Delta x = \frac{1}{40}$ and $\Delta t = \frac{1}{2000}$.
