http://people.sc.fsu.edu/∼jburkardt/presentations/asa_2011_geometry_homework2.pdf

# Homework #13

Algorithms for Science Applications II
Assigned: Friday, 22 April 2011
Due: Friday, 29 April 2011

**This homework will not be assigned! Instead, you should finish up your projects!**

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The numerical values used in the six point quadrature rule are available on Blackboard, or in http://people.sc.fsu.edu/∼jburkardt/latex/asa_2011_geometry_homework/triangle_quadrature_six.m

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**Problem 1:** Quadrature Rules for the Unit Triangle

By the *unit triangle* we mean the triangle **T1** whose vertices are A1 = (1,0), B1 = (0,1), C1 = (0,0). A common problem in calculus involves determining the integral of some function $f(x,y)$ over the unit triangle, which can be symbolized formally as:

$$\int_{T1} f(x,y) = \int_0^1 \int_0^{1-x} f(x,y)\,dy\,dx$$

A *quadrature rule* for the triangle **T1** is a method of approximating integrals by supplying a set of $N$ points $(x1_i, y1_i)$ and corresponding weights $w_i$, where Area(T1) is, of course $\frac{1}{2}$, so that:

$$\int_{T1} f(x,y) \approx \text{Area}(T1) \cdot \sum_{i=1}^{N} w_i\, f(x1_i, y1_i)$$

Here is a quadrature rule for **T1** that uses 3 points:

| W | X1 | Y1 |
|---|---|---|
| $\frac{1}{3}$ | 0 | $\frac{1}{2}$ |
| $\frac{1}{3}$ | $\frac{1}{2}$ | 0 |
| $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

and a quadrature rule for **T1** using 6 points:

| W | X1 | Y1 |
|---|---|---|
| 0.109951743655322 | 0.816847572980459 | 0.091576213509771 |
| 0.109951743655322 | 0.091576213509771 | 0.816847572980459 |
| 0.109951743655322 | 0.091576213509771 | 0.091576213509771 |
| 0.223381589678011 | 0.108103018168070 | 0.445948490915965 |
| 0.223381589678011 | 0.445948490915965 | 0.108103018168070 |
| 0.223381589678011 | 0.445948490915965 | 0.445948490915965 |

Using the 3 point and 6 point quadrature rules, estimate the following integrals over the unit triangle:

- $f(x,y) = 1$ (your answer should be $\frac{1}{2}$!);
- $f(x,y) = x^2 + y^2$;
- $f(x,y) = \sin(xy)$.

**Turn in:** Your two estimates of each integral.

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**Problem 2:** Quadrature Rules for an Arbitrary Triangle

Now suppose that we are interested in approximating the integral of a function over an arbitrary triangle $T$. If we have a quadrature rule with weights W and points (X1,Y1) for the unit triangle $T1$, then we can adapt it for the triangle $T$ by converting each (X1,Y1) point in $T1$ to an (X,Y) point in $T$, using the formula:

- X = A1(x) * X1 + B1(x) * Y1 + C1(x) * ( 1 - X1 - Y1 );
- Y = A1(y) * X1 + B1(y) * Y1 + C1(y) * ( 1 - X1 - Y1 );

Here, $A1(x)$ means the $x$ coordinate of the $A1$ vertex of the $T1$ triangle, and so on.

Our resulting quadrature rule is now:

$$\int_T f(x,y) \approx \text{Area}(T) \cdot \sum_{i=1}^{N} w_i \, f(x_i, y_i)$$

Let the triangle $T$ have vertices $A$=(0,1), $B$=(3,0), $C$=(2,5) and:

- Transform the six point rule from the unit triangle $T1$ to our new triangle $T$;
- Use this rule to estimate the integral of $f(x,y) = x^2$ over $T$.

**Turn in:**

- A table of $X$ and $Y$ for the transformed six point rule over $T$;
- Your estimate for the integral of $f(x,y)$ over $T$.

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